Modern Algebra I, Spring 2017

Homework 8, due Wednesday March 22 before class.
Read Sections 11.1, 11.2, and 5.2. Solve the following problems.

1. Describe all homomorphisms from the group $\mathbb{Z}$ to the group $\mathbb{Z}_4$. Find the kernel and image of each homomorphism. Which of these homomorphisms are surjective?

2. Find all homomorphisms from the group $\mathbb{Z}_6$ to the group $\mathbb{Z}_8$. (Hint: the generator 1 of $\mathbb{Z}_6$ must go to some element m of $\mathbb{Z}_8$. What is the condition on m?) For each homomorphism list its kernel and image.

3. (a) More generally, show that homomorphisms from $\mathbb{Z}_n$ to a group G are in a bijection with elements g of G such that $g^n = 1$, that is, the order of g is a divisor of n.

(b) Use (a) to prove that there's only the trivial homomorphism from $\mathbb{Z}_n$ to $\mathbb{Z}_m$ if $(n, m) = 1$.

4. Use the result in 3(a) to describe all homomorphisms from $\mathbb{Z}_3$ to S_3 and from $\mathbb{Z}_4$ to S_3. Don't forget about the trivial homomorphism!

5. Judson exercise 7abcd in Section 11.3 (page 130).

6. Judson exercise 8 in Section 11.3 (page 130).

7. Judson exercise 19 in Section 5.3 (page 68).

8. Show that the subgroup H of rotations is normal in the dihedral group D_n. Find the quotient group D_n/H.