

Modern algebra I, spring 2017. Quiz 2 Solutions

Check the boxes that are followed by correct statements.

Permutation $(12)(34)$ has order 4.

False: This permutation has order 2.

■ In a cyclic group C_{40} generated by a , element a^5 has order 8.

True: $(a^5)^8 = a^{40} = 1$, and any smaller positive power of a^5 is not 1. In general, if a is a generator of cyclic group of order n then the order of a^k is $\frac{n}{\gcd(n,k)}$. In our case, $n = 40$ and $k = 5$, so we get $\frac{40}{\gcd(40,5)} = 8$.

■ If a group G is commutative and H is a subgroup of G , then H is commutative.

True

The set $\{\text{id}, (123)\}$ is a subgroup of S_3 .

False. This set is not closed under multiplication. $(123)(123) = (132)$, not in the set. Alternatively, the inverse of (123) is not in the set. Either way, it does not satisfy the axioms of a subgroup.

■ The set $\{0, 2, 4\}$ is a subgroup of $\mathbb{Z}/6$. (The group operation on $\mathbb{Z}/6$ is addition.)

True. This subset is closed under the group operation, contains the unit element 0, and contains the additive inverse of each element.

■ For a subgroup H of a group G , any two left cosets g_1H and g_2H are either disjoint or equal.

True. This is a proposition we proved in class.