Modern algebra I, spring 2017. Quiz 3 Solutions

Check the boxes that are followed by correct statements.

☐ If $\psi : G \longrightarrow H$ is a homomorphism and $K \subset G$ a subgroup, then $\psi(K)$ is a subgroup of H.

True: The image of a subgroup of G under a homomorphism is a subgroup of H.

☐ Any subgroup H of $\mathbb{T}$, the group of unit complex numbers under multiplication, is normal in $\mathbb{T}$.

True: $\mathbb{T}$ is an abelian group, and any subgroup of an abelian group is normal.

☐ Any subgroup of the symmetric group S_3 is normal in S_3.

False: For instance, subgroup $\{\mathrm{id}, (12)\}$ is not normal in S_3.

☐ For any two subgroups H, K of a group G, the set

$$HK = \{hk \ : \ h \in H, k \in K\}$$

is a subgroup of G.

False. This is false, in general. For instance, take $H = \{\mathrm{id}, (12)\}$ and $K = \{\mathrm{id}, (13)\}$, both subgroups of S_3. You can check that HK is a 4-element set, and cannot be a subgroup of S_3, which is a group of order 6. If either H of K is normal in G then HK is a subgroup of G (this is part of the Second Isomorphism theorem).

☐ Any subgroup H of a group G is the kernel of a homomorphism from G to some group K.

False: If H is not normal, it cannot be the kernel of a homomorphism into any group. True for normal subgroups.

☐ Any homomorphism from $\mathbb{Z}$ to $\mathbb{Z}_6$ is surjective.

False: For instance, the trivial homomorphism $\mathbb{Z} \longrightarrow \mathbb{Z}_6$ is not surjective. There are three other homomorphisms from $\mathbb{Z}$ to $\mathbb{Z}_6$ that are not surjective, can you find them all?