

CALCULUS III: HW 10

Due Tuesday, November 30 by 11pm on Gradescope. Please show all of your work, typed or handwritten clearly and legibly. When you upload your solutions to Gradescope, be sure to select the pages that each question is on.

QUESTION 1

Let $f(x, y, z) = \ln(1 + 2z(x^2 - y^2))$.

- (a). Find the gradient of f at the point $(-2, 2, 2)$.
- (b). Find the direction in which f increases most rapidly at $(-2, 2, 2)$. What is this maximal rate of increase?

QUESTION 2

Let $f(x, y) = \sqrt{x^2 - y^2}$.

- (a). Find the directional derivative of f at the point $(-4, 3)$ in the direction of the vector $\langle 1, 2 \rangle$.
- (b). In which direction does f increase most quickly at the point $(-4, 3)$? What is this maximal rate of increase?

QUESTION 3

Is there a point on the sphere $x^2 + y^2 + z^2 = 4$ at which the tangent plane is parallel to the plane $2x - 2y + z = 10$? If so, find it.

QUESTION 4

Suppose that the temperature at a point (x, y, z) is given by $T(x, y, z) = 100 + \cos(\pi x) - y^2 - z^2$.

- (a). Find the rate of change of the temperature at the point $(1, 2, 1)$ in the direction of the vector $\langle -1, -2, 2 \rangle$.
- (b). At the point $(1, 2, 1)$, in which direction does the temperature increase most quickly? What is that maximum rate of change of temperature?

QUESTION 5

Find equations for the tangent plane and normal line to the surface $x^2 + y^2 = 2 \tan(z)$ at the point $(1, -1, 5\pi/4)$.

QUESTION 6

Consider the cone $z^2 = x^2 + y^2$ and the sphere $x^2 + y^2 + z^2 = 18$. They intersect at the point $(0, 3, 3)$, among others. Find the angle (or the cosine of the angle) between these surfaces at the point $(0, 3, 3)$.

Note: the angle between two surfaces is the same as the angle between their normal lines.