CALCULUS III: HW 11

Due Tuesday, December 7 by 11pm on Gradescope. Please show all of your work, typed or handwritten clearly and legibly. When you upload your solutions to Gradescope, be sure to select the pages that each question is on.

Question 1

Let

$$f(x, y) = 2x^4 - 2xy + y^2 - 2$$

Find the local minimum points, local maximum points, and saddle points of f.

Question 2

Find three positive numbers whose sum is 15 and whose product is as large as possible.

Question 3

Find the points on the surface $z^2 = xy + 4$ which are closest to the origin. (Hint: express z^2 in terms of x and y, and use this to express the distance formula as a function of x and y. Then minimize this distance function, or its square.)

Question 4

Let

$$f(x, y) = 5 + 2xy + \frac{20}{x} + \frac{25}{y}$$

Find the local minimum points, local maximum points, and saddle points of f.

Question 5

Let

$$f(x, y) = x^2 - 2x + \frac{y^2}{2} - 2y.$$

(a). Find the local minimum points, local maximum points, and saddle points of f.
(b). Sketch the region

$$D = \{(x, y) : x \geq 0, y \geq 0, y \leq 4 - x\}$$

and find its boundary.
(c). Find the absolute minimum and absolute maximum values of f on D.

Question 6

Find the absolute maximum and absolute minimum values of the function $f(x, y) = x^3 - 3x + y^3 - 3y$ on the region

$$D = \{(x, y) : -2 \leq x \leq 2, -2 \leq y \leq 2\}$$

Question 7

Find the absolute maximum and absolute minimum values of the function $f(x, y) = x^2 + y^2 + xy - 3y$ on the closed half-disk

$$D = \{(x, y) : -\sqrt{9 - x^2} \leq y \leq 0\}.$$

Hint: the boundary of this half-disk will consist of two parts.