

CALCULUS III: HW 12

Due Tuesday, December 14 by 11pm on Gradescope. Please show all of your work, typed or handwritten clearly and legibly. When you upload your solutions to Gradescope, be sure to select the pages that each question is on.

QUESTION 1

Let $f(x, y) = x^2 + 2xy + y^2$. Find the maximum and minimum values of f subject to the constraint $x^2 + 4y^2 = 20$. Be sure to specify the points at which these values are obtained.

QUESTION 2

The sphere $x^2 + y^2 + z^2 = 9$ and the plane $2x + 2y + 2z = -10$ intersect in an ellipse. Use Lagrange multipliers, or any other method, to find the lowest point (= point with minimal z -value) on this ellipse.

QUESTION 3

Find the absolute maximum and absolute minimum values of the function $f(x, y) = x^3 + y^3 - 3y$ on the region $\{(x, y) \in \mathbb{R}^2 : x^2 + (y - 1)^2 \leq 4\}$. Hint: to find the maximum and minimum on the **boundary** of this region, you might want to use Lagrange multipliers.

QUESTION 4

Let $f(x, y, z) = 2x^2 + y^2 + z^2$. Find the minimum value of f subject to the constraint $2x + y + z = 4$.

QUESTION 5

Find all complex solutions to the equation $w^3 = -27$ (that is, find all of the third roots of -27 .) Express your answers in the form $a + bi$.

QUESTION 6

Find all solutions in complex numbers to the equation $w^2 - 3w + 5 = 0$.

QUESTION 7

Find the complex number

$$\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)^{15}.$$

Express your answer in the form $a + bi$.

QUESTION 8

Find all solutions in complex numbers to the equation $w^4 + 3w^2 + 2 = 0$.