

CALCULUS III: HW 2

Due Tuesday, September 21 by 11pm on Gradescope. Please show all of your work, typed or handwritten clearly and legibly.

QUESTION 1

Let $P = (1, 2, 3)$ and $Q = (-1, 4, 4)$. Find the vector from P to Q and its length.

QUESTION 2

For each of the following vectors, find their magnitude (aka length) and find a unit vector in the same direction.

(a). $\mathbf{v} = -3\mathbf{i} + 2\mathbf{j} - 6\mathbf{k}$

(b). $\mathbf{v} = \langle 1, -2, -2 \rangle$

QUESTION 3

Find all values of t such that $\langle 2/3, -1/3, t \rangle$ is a unit vector. If there are no such values, explain why.

QUESTION 4

Find the angles of the triangle whose vertices are $P = (-2, -1, -4)$, $Q = (1, -1, -3)$, and $R = (0, -1, 0)$.

QUESTION 5

Let $P = (3, 1, -2)$, $Q = (1, t + 1, 2)$, and $R = (11, t + 1, 4)$. For which values of t will the angle between the vectors \mathbf{PQ} and \mathbf{PR}

(a). be a right angle? If there are no such values, explain why.

(b). be an acute angle (i.e. less than 90 degrees)? If there are no such values, explain why.

QUESTION 6

Find the scalar projection and vector projection of $\mathbf{b} = \langle -3, 7, 9 \rangle$ onto $\mathbf{a} = \langle 3, 0, -4 \rangle$.

QUESTION 7

Let $A = (-2, -3, 1)$ and $B = (4, -3, -7)$. Find an equation for the sphere consisting of all points $P = (x, y, z)$ such that the vector \mathbf{PA} is orthogonal to the vector \mathbf{PB} . What are its center and radius?

QUESTION 8

Find $\mathbf{a} \times \mathbf{b}$ if

(a). $\mathbf{a} = \langle 0, 2, 1 \rangle$, $\mathbf{b} = \langle -3, -6, 12 \rangle$

(b). $\mathbf{a} = \langle -1, 3, 3 \rangle$, $\mathbf{b} = \langle 0, -4, 6 \rangle$.

QUESTION 9

Find the area of the triangle with vertices $P = (-2, -1, -4)$, $Q = (1, -1, -3)$, and $R = (0, -1, 0)$.

QUESTION 10

- (a). Find a vector \mathbf{v} such that $\langle 1, 0, -1 \rangle \times \mathbf{v} = \langle 1, 2, 3 \rangle$, or explain why there is no such vector.
- (b). Find a vector \mathbf{v} such that $\langle -3, 0, 1 \rangle \times \mathbf{v} = \langle 1, 2, 3 \rangle$, or explain why there is no such vector.

QUESTION 11

Consider the points $A = (0, -1, 3)$, $B = (2, 0, 2)$, $C = (1, -4, 5)$, and $D = (-1, -2, 2)$. Find the volume of the parallelepiped with adjacent edges AB , AC , and AD .