

CALCULUS III: HW 4

Due Tuesday, October 12 by 11pm on Gradescope. Please show all of your work, typed or handwritten clearly and legibly.

QUESTION 1

Find an equation for the hyperbola H with focus points $F_1 = (0, -5)$ and $F_2 = (0, 5)$, and asymptotes $y = \frac{4}{3}x$ and $y = -\frac{4}{3}x$.

QUESTION 2

Find parametric equations for the ellipse

$$\frac{(x-1)^2}{9} + \frac{(y+3)^2}{16} = 1$$

QUESTION 3

Find the focus, vertex, and directrix of the parabola with equation $y^2 = -16x$. Sketch the parabola.

QUESTION 4

- (a). Sketch the hyperbola $4x^2 - y^2 = 4$. Find its vertices, focus points, and asymptotes.
(b). Find the intersection points of the hyperbola in part (a) with the ellipse $4x^2 + y^2 = 4$.

QUESTION 5

Sketch the hyperbola

$$\frac{y^2}{100} - \frac{x^2}{36} = 1$$

Find its vertices, focus points, and asymptotes.

QUESTION 6

Find an equation for the ellipse E with focus points $F_1 = (-12, 0)$ and $F_2 = (12, 0)$, and which consists of all points $P = (x, y)$ whose sum of distances to these focus points is equal to 26. That is, E consists of all points $P = (x, y)$ such that $|PF_1| + |PF_2| = 26$.

QUESTION 7

Consider the quadric surface $z = x^2 - 4y^2 + 4$.

- (a). Find and sketch its $y = k$ traces for $k = 0$ and $k = 1$.
(b). Find and sketch its $z = k$ traces for $k = 3$, $k = 4$, and $k = 5$.

QUESTION 8

Consider the quadric surface $x^2 - y^2 + 4z^2 = 0$.

- (a). Find and sketch its $y = k$ and $z = k$ traces for $k = 0$ and $k = 1$.
- (b). Which quadric surface is this?

QUESTION 9

Consider the quadric surface $x^2 + 4x + y^2 + 2y - z = 0$.

- (a). Find and sketch its $x = k$ traces for $k = 0$ and $k = 1$.
- (b). Find and sketch its $z = k$ traces for $k = -2$, $k = -1$, and $k = 0$.
- (c). Which quadric surface is this?

QUESTION 10

The surface with equation $-x^2 + y^2 - 9z^2 = 1$ is a quadric surface. Find and sketch its traces in the planes $y = 0$, $y = 1$, and $y = 2$.

QUESTION 11

Consider the quadric surface $-4x^2 + 4y^2 + z^2 = 16$.

- (a). Find and sketch its $z = k$ traces for $k = 3$, $k = 4$, and $k = 5$.
- (b). Find and sketch its $x = k$ traces for $k = 0$ and $k = 1$.
- (c). Which quadric surface is this?

QUESTION 12

Find an equation for the surface consisting of all points $P = (x, y, z)$ whose distance to the plane $x = 2$ is twice their distance to the point $(-1, 3, 1)$. Which quadric surface is this?