CALCULUS III: HW 5

Due Tuesday, October 19 by 11pm on Gradescope. Please show all of your work, typed or handwritten clearly and legibly. When you upload your solutions to Gradescope, be sure to select the pages that each question is on.

Question 1

Find the domain of the vector function $\mathbf{r}(t) =< \ln(2t-3), e^{t^2}, \frac{1}{t^2-4} >$.

Question 2

Does the limit

$$\lim_{t\to 0} < t^4 cos(\frac{1}{t}), \ln(t+3), \frac{3t}{e^t} >$$

exist? If so, find it.

Question 3

At what point(s) does the curve $\mathbf{r}(t) =< t\cos(t), t-1, , t\sin(t)+1 >$ intersect the sphere $x^2 + (y+1)^2 + (z-1)^2 = 2$?

Question 4

An airplane's position vector at time t is described by the vector function $\mathbf{r}_1(t) =< t, t^2 - 6t + 8, t^3 - 8 >$. A flying saucer's position vector at time t is described by the vector function $\mathbf{r}_2(t) =< t+1, t^2 - t, t^3 - 1 >$.
(a). Do the plane and the flying saucer collide? That is, are they ever in the same spot at the same time?
(b). Do the paths of the airplane and the flying saucer intersect? That is, are they ever in the same spot at possibly different times?

Question 5

Find a vector function that represents the curve of intersection of the parabolic cylinder $y = 1 - z^2$ and the elliptic paraboloid $x = y^2 + 2z^2$.

Question 6

Let $\mathbf{r}(t) =< 6\sin(t), 8\sin(t), 10\cos(t) >$. Find an equation for a sphere that this curve lies on.

Question 7

Let $\mathbf{v}(t) =< \frac{1}{t+1}, 2t, 3\sin(3t) >$.
(a). Find an equation (vector or parametric) for the tangent line to this curve at $t = 1$.
(b). Find an antiderivative $\mathbf{r}(t)$ to $\mathbf{v}(t)$ such that $\mathbf{r}(0) =< 0, 1, -3 >$.

Question 8

If $\mathbf{r}(0) =< 0, -4, 0 >$ and $\mathbf{r}'(t) =< -e^t, 3t^2, -2\cos(2t) >$, find $\mathbf{r}(t)$.

Question 9

Let $\mathbf{r}(t) =< 1 - 3sin(t), e^t - 2t, -t^2 >$.
(a). Find an equation (vector or parametric) for the tangent line to this curve at $t = 0$.
(b). Find $\mathbf{r}''(t)$.