CALCULUS III: HW 6

Due Tuesday, October 26 by 11pm on Gradescope. Please show all of your work, typed or handwritten clearly and legibly. When you upload your solutions to Gradescope, be sure to select the pages that each question is on.

Question 1

Find the length of the following curve from $t = 1$ to $t = 3$.

$$\mathbf{r}(t) =< 18t, 4t^{3/2}, \frac{1}{2}t^2 >$$

Question 2

Find the curvature of the curve $\mathbf{r}(t) =< -t^3, 2t^2, 3t >$ at $t = 0$.

Question 3

Find the curvature of the curve $\mathbf{r}(t) =< 2\cos(t), -2\sin(t), -t >$ at $t = \pi/2$.

Question 4

Find the osculating plane of the curve $\mathbf{r}(t) =< 3\cos(t), 4\cos(t), -5\sin(t) >$ at the point $(-3, -4, 0)$.

Question 5

Let $\mathbf{r}(t) =< 1 + t^2, t^3, e^t >$. At which point(s) is the normal plane of this curve parallel to the plane $2x + 3y + ez = 0$?

Question 6

A projectile is released from (0,0) with an initial speed of 50 m/s at an elevation angle of 60 degrees. Assume, for ease of calculation, that the acceleration due to gravity is 10 m/s^2.
(a). Find the position and velocity functions of the projectile.
(b). At what times is the projectile at a height three quarters of its maximum height?

Question 7

An object moves in space with position function

$$\mathbf{r}(t) =< 2t + 1, t^2, \frac{1}{3}t^3 - 1 >$$

(a). Find its velocity and acceleration functions.
(b). At which point in $\mathbb{R}^3$ is its speed minimal?

Question 8

An object of mass 0.5 moves with position function $\mathbf{r}(t) =< 3\cos(t), 4\cos(t), -5\sin(t) >$.

(a). Find the speed of this object.
(b). Find the force vector acting on this object at time $t = \pi$. Show that it points from the object towards the origin. That is, show that it is in the same direction as the vector from the object to $(0, 0, 0)$.
(c). Find the object's tangential and normal components of acceleration at time t.

Question 9

A particle moves with position function $\mathbf{r}(t) =< -t^3, 2t^2, 3t >$. Find its tangential and normal components of acceleration at $t = 0$.