

CALCULUS III: HW 7

Due Wednesday, November 3 by 11pm on Gradescope. Please show all of your work, typed or handwritten clearly and legibly. When you upload your solutions to Gradescope, be sure to select the pages that each question is on.

QUESTION 1

Let $f(x, y) = xy + 1$. Describe and sketch the level curves $f(x, y) = 0$, $f(x, y) = 1$, $f(x, y) = 2$, and $f(x, y) = 3$. You may draw all of them in the same plane.

QUESTION 2

Let $f(x, y) = \sqrt{16 - 4x^2 - y^2}$.

- Find and sketch the domain of this function.
- Describe and sketch the level curves $f(x, y) = 0$, $f(x, y) = 2$, and $f(x, y) = 4$. You may draw all of them in the same plane.
- Find the range of this function.

QUESTION 3

Let $f(x, y) = \frac{2x}{x^2 - y^2}$.

- Find the domain of f .
- Describe and sketch the level curves of $f(x, y) = -1$, $f(x, y) = 0$, and $f(x, y) = 1$. You may draw all of them in the same plane.

QUESTION 4

Compute the following limit, or show that it doesn't exist.

$$\lim_{(x,y) \rightarrow (0,0)} \frac{3x^2y^4}{x^4 + y^4}$$

QUESTION 5

Compute the following limit, or show that it doesn't exist.

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 \sin^2(y)}{x^4 + y^4}$$

QUESTION 6

Compute the following limit, or show that it doesn't exist.

$$\lim_{(x,y) \rightarrow (1,0)} \frac{\cos(\sqrt{(x-1)^2 + y^2}) - 1}{(x-1)^2 + y^2}$$

QUESTION 7

Let $f(x, y) = \sqrt{4 - x^2 - y^2}$.

- Calculate its first partial derivatives f_x and f_y . What is the domain of the function f_x ?

(b). Calculate its second partial derivatives f_{xx} , f_{yy} , f_{xy} , and f_{yx} .

QUESTION 8

(a). For which real number a is there a two-variable function $f(x, y)$ such that $f_x = 2xy + 2y^2$ and $f_y = 3y^2 + axy + x^2$? (Hint: use Clairaut's theorem about f_{xy} and f_{yx} .)

(b). (Optional; extra credit, 5 points.) For the value of a you found above, find a function $f(x, y)$ such that $f_x = 2xy + 2y^2$ and $f_y = 3y^2 + axy + x^2$.

QUESTION 9

Let z be defined implicitly as a function of x and y by the equation

$$\frac{1}{x} + \frac{1}{y^2} + \frac{1}{z^2} = 1$$

Find $\partial z / \partial x$ and $\partial z / \partial y$ at the point $(2, 2, 2)$.