CALCULUS III: HW 8

Due Tuesday, November 16 by 11pm on Gradescope. Please show all of your work, typed or handwritten clearly and legibly. When you upload your solutions to Gradescope, be sure to select the pages that each question is on.

Question 1

Let $f(x, y) = \sin(9x^2 - y^2)$. Find an equation for the tangent plane to the graph of f at the point $(1, -3, 0)$.

Question 2

Let $f(x, y) = \ln(4 - x^2 - y^2)$.
(a). Explain why f is differentiable at the point where $(x, y) = (0, 1)$.
(b). Find an equation for the tangent plane to the graph of f at the point where $(x, y) = (0, 1)$.
(c). Find an equation for the tangent plane to the graph of f at the point where $(x, y) = (0, 0)$.

Question 3

Let $f(x, y) = \sqrt{x^3 + y^3}$.
(a). Explain why f is differentiable at the point where $(x, y) = (1, 2)$.
(b). Find a linear approximation for $f(x, y)$ near $(1, 2)$, and use it to approximate the number $\sqrt{0.99^3 + 2.02^3}$.

Question 4

Find a point on the surface

$$z = \ln(x) + y^2$$

where the tangent plane is parallel to the plane $2x + 6y - 2z = 0$.

Question 5

A can has the shape of an elliptic cylinder, with base radii a and b, and height h. Its volume is given by $V(a, b, h) = \pi abh$.
(a). Find the differential dV in terms of da, db, and dh.
(b). A cylindrical can is measured to have height $h = 8$ centimeters, and radii $a = 3$ cm and $b = 4$ cm. The error for each of these measurements is at most 0.1 cm.. Use differentials to estimate the maximal error in measuring the volume of the can.

Question 6

Suppose there is a surface S whose equation we don't know, but we want to find an equation for the tangent plane to S at the point $(1, 0, 1)$. We do know that the curves with vector equations

$$\mathbf{r}_1(t) =< \cos(t), -3\sin(t), 1 - t >$$

and

$$\mathbf{r}_2(s) =< 1 + \sin(s), \cos(s) - 1, 2s^2 + 1 >$$

lie on the surface S.
(a). Find equations for the tangent lines to $\mathbf{r}_1(t)$ and to $\mathbf{r}_2(s)$ at the point $(1, 0, 1)$.
(b). Find an equation for the tangent plane to the surface S at the point $(1, 0, 1)$. (Hint: the tangent plane contains both of the lines that you found in part (a).)

Question 7

Let $f(x, y) = \frac{x}{\sin(y)}$.
(a). Find the domain of f.
(b). Find f_x and f_y. Where is f differentiable?
(c). Find a linear approximation to $f(x, y)$ near the point where $(x, y) = (1, \pi/2)$.

Question 8

Let $f(x, y) = \int_x^y e^{-2t^2}\, dt$.
(a). Find f_x and f_y. (Hint: use the fundamental theorem of calculus.)
(b). Find an equation for the tangent plane to the graph of f at the point $(-1, -1, 0)$.