CALCULUS III: HW 9

Question 1

Suppose that $z = ycos(x) - x^3$, $x = s + t - 1$, $y = st - t^2$. Find $\partial z/\partial s$ and $\partial z/\partial t$ at the point $(s, t) = (0, 1)$.

Question 2

Suppose that $z = f(x, y)$, $x = st$ and $y = s + t^2$. Suppose that $\partial z/\partial x(x = 2, y = 5) = 3$ and that $\partial z/\partial y(x = 2, y = 5) = -2$. Find $\partial z/\partial s$ and $\partial z/\partial t$ at the point $(s, t) = (1, 2)$.

Question 3

In a simple electrical circuit, the current I can be expressed as a function of the resistance R and the voltage V by $I(R, V) = V/R$. At a certain time, the resistance is 200 and the voltage is 0.4. At that specific time, $dR/dt = 0.1$ and $dV/dt = -0.05$. Find the rate of change of the current at that time.

Question 4

Assume z is implicitly defined as a function of x and y by the equation

$$x^2 + y^2 - z^2 = 0$$

Find $\partial z/\partial x$ and $\partial z/\partial y$ at the point $(3, -4, 5)$.

Question 5

Let $f(x, y) = \int_{y^2}^{x^2} e^{-2t^2} dt$. Find f_x and f_y. Hint: Use the Fundamental Theorem of Calculus and the Chain Rule.

Question 6

Let $f(x, y) = \frac{-2y}{x^2+y^2}$.

(a). Find the gradient of f at the point where $(x, y) = (1, 2)$.

(b). Find the directional derivative of f at the point where $(x, y) = (1, 2)$ in the direction of the vector $< 3, -4 >$.

Question 7

In Figure 1 below you will find a contour map of a function $f(x, y)$. The positive x direction is to the right, and the positive y direction is up.

(a). Is $f_x(125, 50)$ positive or negative? Justify your answer.

(b). Is $f_y(50, 75)$ positive or negative? Justify your answer.

(c). Is the directional derivative of f at the point where $(x, y) = (0, -25)$ in the direction of the vector $< -1, -1 >$ positive or negative? Justify your answer.

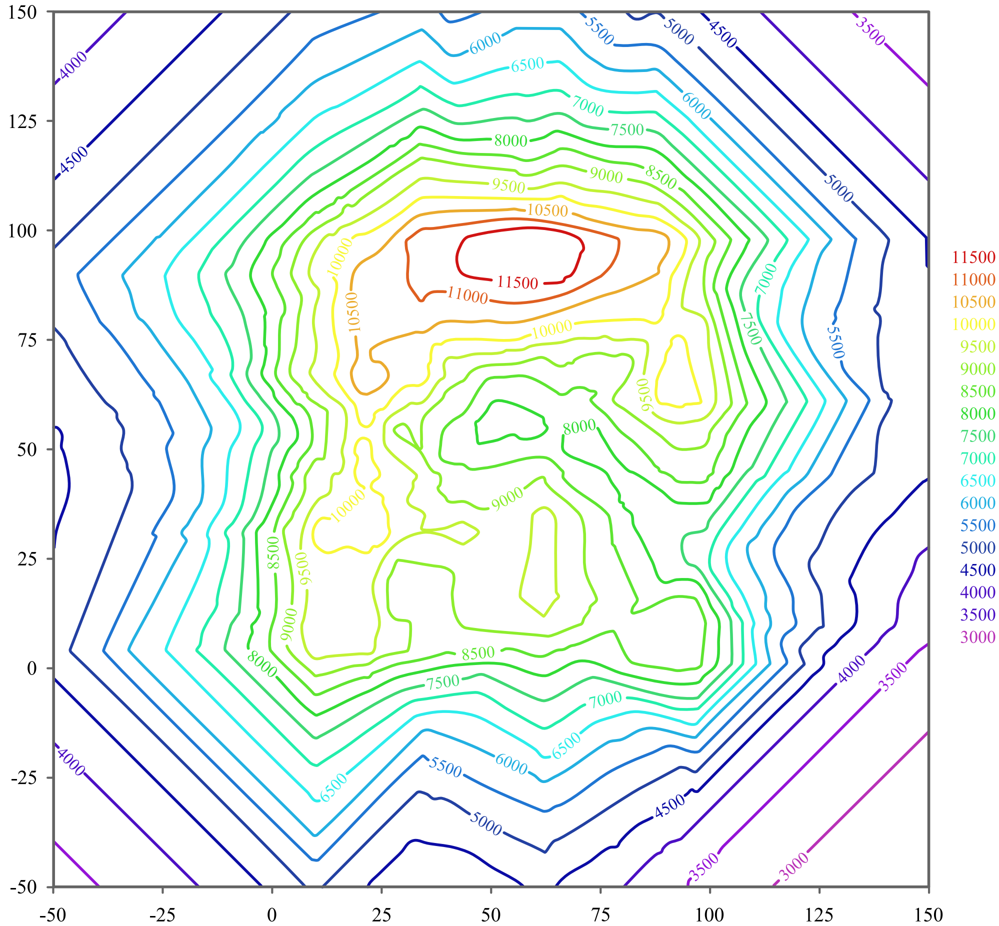

FIGURE 1. Contour map of f from question 7

QUESTION 8

Let $f(x, y, z) = \sqrt{9 - 2x^2 - y^2 - z^2}$.
(a). Find the gradient of f at the point where $(x, y, z) = (0, 1, 2)$.
(b). Find the directional derivative of f at the point $(x, y, z) = (0, 1, 2)$ in the direction of the vector $< 1, -2, 2 >$.