

Calculations in Monopole Floer Homology

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Disclosure: \mathbb{Z} may actually denote the integers, the real, or the complex numbers as necessary. I expect that this disclosure is not actually necessary.

1 Zero surgery on the Whitehead Link, L_5a_1

To obtain zero surgery on the Whitehead Link we do surgery on the Borromean rings in Figure 1 where the green (A) and blue (B) components are 0 framed and the red (C) is -1 framed. Call this 3-manifold Wh or Y_{-1} . Now we apply the $-1 \rightarrow 0 \rightarrow \infty$ surgery exact sequence to the red curve to compute the Monopole Homology. Note that $Y_0 \cong T^3$ and $Y_\infty \cong 2S^1 \times S^2$. We compute that $W_{0\infty}$ induces a grading shift of 0 and that the map on \overline{HM} is induced from:

$$\begin{array}{ccc}
 T^3 & & 2S^1 \times S^2 \\
 \text{even} & & \text{even} \\
 a & & a \wedge b \\
 b & \longrightarrow & \text{pt} \\
 c & \longrightarrow & \\
 \text{odd} & & \text{odd} \\
 a \wedge b & \longrightarrow & a \\
 a \wedge c & \longrightarrow & b \\
 b \wedge c & \longrightarrow &
 \end{array}$$

Also, we compute that $W_{\infty-1}$ induces a grading shift of 0.

In all non-torsion $spin^{\mathbb{C}}$ structures the exact-perturbation groups for T^3 and $nS^1 \times S^2$ are all zero. Therefore, the same holds for zero surgery on the Whitehead link and the computation below is done solely in the trivial $spin^{\mathbb{C}}$ structure, \mathfrak{s}_0 .

α		β
Kernel		Cokernel
0	$0 \longleftarrow \widehat{HM}_1(T^3) \xrightarrow{0} \widehat{HM}_1(2S^1 \times S^2) \longrightarrow 0$	0
\mathbb{Z}^2	$\downarrow \mathbb{Z}^3 \longleftarrow \overline{HM}_0(T^3) \xrightarrow{\text{rk}=1} \overline{HM}_0(2S^1 \times S^2) \longrightarrow \mathbb{Z}^2$	\mathbb{Z}
\mathbb{Z}^2	$\downarrow \mathbb{Z}^3 \longleftarrow \widetilde{HM}_0(T^3) \xrightarrow{\text{rk}=1} \widetilde{HM}_0(2S^1 \times S^2) \longrightarrow \mathbb{Z}^2$	\mathbb{Z}
0	$\downarrow 0 \longleftarrow \widehat{HM}_0(T^3) \xrightarrow{0} \widehat{HM}_0(2S^1 \times S^2) \longrightarrow 0$	0
\mathbb{Z}	$\downarrow \mathbb{Z}^3 \longleftarrow \overline{HM}_{-1}(T^3) \xrightarrow{\text{rk}=2} \overline{HM}_{-1}(2S^1 \times S^2) \longrightarrow \mathbb{Z}^2$	0
\mathbb{Z}	$\downarrow \mathbb{Z}^3 \longleftarrow \widetilde{HM}_{-1}(T^3) \xrightarrow{\text{rk}=2} \widetilde{HM}_{-1}(2S^1 \times S^2) \longrightarrow \mathbb{Z}^2$	0
0	$\downarrow 0 \longleftarrow \widehat{HM}_{-1}(T^3) \xrightarrow{0} \widehat{HM}_{-1}(2S^1 \times S^2) \longrightarrow \mathbb{Z}$	\mathbb{Z}
\mathbb{Z}^2	$\downarrow \mathbb{Z}^3 \longleftarrow \overline{HM}_{-2}(T^3) \xrightarrow{\text{rk}=1} \overline{HM}_{-2}(2S^1 \times S^2) \longrightarrow \mathbb{Z}^2$	\mathbb{Z}
\mathbb{Z}^3	$\downarrow \mathbb{Z}^3 \longleftarrow \widetilde{HM}_{-2}(T^3) \xrightarrow{0} \widetilde{HM}_{-2}(2S^1 \times S^2) \longrightarrow \mathbb{Z}$	\mathbb{Z}
\mathbb{Z}	$\downarrow \mathbb{Z}^3 \longleftarrow \widehat{HM}_{-2}(T^3) \xrightarrow{\text{rk}=2} \widehat{HM}_{-2}(2S^1 \times S^2) \longrightarrow \mathbb{Z}^2$	0
\mathbb{Z}	$\downarrow \mathbb{Z}^3 \longleftarrow \overline{HM}_{-3}(T^3) \xrightarrow{\text{rk}=2} \overline{HM}_{-3}(2S^1 \times S^2) \longrightarrow \mathbb{Z}^2$	0
0	$\downarrow 0 \longleftarrow \widetilde{HM}_{-3}(T^3) \xrightarrow{0} \widetilde{HM}_{-3}(2S^1 \times S^2) \longrightarrow 0$	0
\mathbb{Z}^2	$\downarrow \mathbb{Z}^3 \longleftarrow \widehat{HM}_{-3}(T^3) \xrightarrow{\text{rk}=1} \widehat{HM}_{-3}(2S^1 \times S^2) \longrightarrow \mathbb{Z}^2$	\mathbb{Z}
\mathbb{Z}^2	$\downarrow \mathbb{Z}^3 \longleftarrow \overline{HM}_{-4}(T^3) \xrightarrow{\text{rk}=1} \overline{HM}_{-4}(2S^1 \times S^2) \longrightarrow \mathbb{Z}^2$	\mathbb{Z}
0	$\downarrow 0 \longleftarrow \widetilde{HM}_{-4}(T^3) \xrightarrow{0} \widetilde{HM}_{-4}(2S^1 \times S^2) \longrightarrow 0$	0

	$\overline{0} \xrightarrow{i} \check{0} \xrightarrow{j} \widehat{0} \xrightarrow{p} \overline{-1} \xrightarrow{i} \check{-1} \xrightarrow{j} \widehat{-1} \xrightarrow{p} \overline{-2} \xrightarrow{i} \check{-2} \xrightarrow{j} \widehat{-2} \xrightarrow{p} \overline{-3} \xrightarrow{i} \check{-3}$
β	$\mathbb{Z} \quad \mathbb{Z} \quad 0 \quad 0 \quad 0 \quad \mathbb{Z} \quad \mathbb{Z} \quad \mathbb{Z} \quad 0 \quad 0 \quad 0$
\oplus	$\oplus \xrightarrow{\text{rk}=2} \oplus \xrightarrow{0} \oplus \xrightarrow{0} \oplus \xrightarrow{\text{rk}=2} \oplus \xrightarrow{\text{rk}=1} \oplus \xrightarrow{\text{rk}=1} \oplus \xrightarrow{\text{rk}=1} \oplus \xrightarrow{0} \oplus \xrightarrow{\text{rk}=2} \oplus \xrightarrow{0} \oplus$
$\alpha[1]$	$\mathbb{Z} \quad \mathbb{Z} \quad 0 \quad \mathbb{Z}^2 \quad \mathbb{Z}^3 \quad \mathbb{Z} \quad \mathbb{Z} \quad 0 \quad \mathbb{Z}^2 \quad \mathbb{Z}^2 \quad 0$

Outside of this range, the homology is standard. (i.e. isomorphic to the homology of $2S^1 \times S^2$, which has the same first homology & triple intersection product as Wh .) So $HM_{-1}(Wh, \mathfrak{s}_0) = \mathbb{Z}$ and $HM_*(Wh, \mathfrak{s}_n) = 0$ otherwise.

2 Zero surgery on the 2–twist Whitehead Link,

L_7a_4

To obtain zero surgery on the 2–twist Whitehead Link we do surgery on a variant of the Borromean rings in Figure 2 where the blue (A) and orange (B) components are 0 framed and the green (C) and red (D) are -1 framed. Call this 3–manifold $Wh(2)$ or Y_{-1} . Now we apply the $-1 \rightarrow 0 \rightarrow \infty$ surgery exact sequence to the red curve to compute the Monopole Homology. Note that $Y_0 \cong T^3$ and $Y_\infty \cong Wh$. We compute that $W_{0\infty}$ induces a grading shift of 0 and that the map on \overline{HM} is induced from:

$$\begin{array}{ccc}
 T^3 & & Wh \\
 \text{even} & & \text{even} \\
 a & & a \wedge b \\
 b & \nearrow & \text{pt} \\
 c & & \\
 \text{odd} & & \text{odd} \\
 a \wedge b & \nearrow & a \\
 a \wedge c & \nearrow & b \\
 b \wedge c & \nearrow &
 \end{array}$$

Also, we compute that $W_{\infty-1}$ induces a grading shift of 0.

In all non-torsion $spin^{\mathbb{C}}$ structures the exact-perturbation groups for T^3 and Wh are all zero. Therefore, the same holds for zero surgery on the 2–twist Whitehead link and the computation below is done solely in the trivial $spin^{\mathbb{C}}$ structure, \mathfrak{s}_0 .

α		β
Kernel		Cokernel
0	$0 \xlongequal{\quad} \widehat{HM}_1(T^3) \xrightarrow{0} \widehat{HM}_1(Wh) \xlongequal{\quad} 0$	0
\mathbb{Z}^2	\downarrow $\mathbb{Z}^3 \xlongequal{\quad} \overline{HM}_0(T^3) \xrightarrow{\text{rk}=1} \overline{HM}_0(Wh) \xlongequal{\quad} \mathbb{Z}^2$	\mathbb{Z}
\mathbb{Z}^2	\downarrow $\mathbb{Z}^3 \xlongequal{\quad} \widetilde{HM}_0(T^3) \xrightarrow{\text{rk}=1} \widetilde{HM}_0(Wh) \xlongequal{\quad} \mathbb{Z}^2$	\mathbb{Z}
0	\downarrow $0 \xlongequal{\quad} \widehat{HM}_0(T^3) \xrightarrow{0} \widehat{HM}_0(Wh) \xlongequal{\quad} 0$	0
\mathbb{Z}	\downarrow $\mathbb{Z}^3 \xlongequal{\quad} \overline{HM}_{-1}(T^3) \xrightarrow{\text{rk}=2} \overline{HM}_{-1}(Wh) \xlongequal{\quad} \mathbb{Z}^2$	0
\mathbb{Z}	\downarrow $\mathbb{Z}^3 \xlongequal{\quad} \widetilde{HM}_{-1}(T^3) \xrightarrow{\text{rk}=2} \widetilde{HM}_{-1}(Wh) \xlongequal{\quad} \mathbb{Z}^3$	\mathbb{Z}
0	\downarrow $0 \xlongequal{\quad} \widehat{HM}_{-1}(T^3) \xrightarrow{0} \widehat{HM}_{-1}(Wh) \xlongequal{\quad} \mathbb{Z}^2$	\mathbb{Z}^2
\mathbb{Z}^2	\downarrow $\mathbb{Z}^3 \xlongequal{\quad} \overline{HM}_{-2}(T^3) \xrightarrow{\text{rk}=1} \overline{HM}_{-2}(Wh) \xlongequal{\quad} \mathbb{Z}^2$	\mathbb{Z}
\mathbb{Z}^3	\downarrow $\mathbb{Z}^3 \xlongequal{\quad} \widetilde{HM}_{-2}(T^3) \xrightarrow{0} \widetilde{HM}_{-2}(Wh) \xlongequal{\quad} \mathbb{Z}$	\mathbb{Z}
\mathbb{Z}	\downarrow $\mathbb{Z}^3 \xlongequal{\quad} \widehat{HM}_{-2}(T^3) \xrightarrow{\text{rk}=2} \widehat{HM}_{-2}(Wh) \xlongequal{\quad} \mathbb{Z}^2$	0
\mathbb{Z}	\downarrow $\mathbb{Z}^3 \xlongequal{\quad} \overline{HM}_{-3}(T^3) \xrightarrow{\text{rk}=2} \overline{HM}_{-3}(Wh) \xlongequal{\quad} \mathbb{Z}^2$	0
0	\downarrow $0 \xlongequal{\quad} \widetilde{HM}_{-3}(T^3) \xrightarrow{0} \widetilde{HM}_{-3}(Wh) \xlongequal{\quad} 0$	0
\mathbb{Z}^2	\downarrow $\mathbb{Z}^3 \xlongequal{\quad} \widehat{HM}_{-3}(T^3) \xrightarrow{\text{rk}=1} \widehat{HM}_{-3}(Wh) \xlongequal{\quad} \mathbb{Z}^2$	\mathbb{Z}
\mathbb{Z}^2	\downarrow $\mathbb{Z}^3 \xlongequal{\quad} \overline{HM}_{-4}(T^3) \xrightarrow{\text{rk}=1} \overline{HM}_{-4}(Wh) \xlongequal{\quad} \mathbb{Z}^2$	\mathbb{Z}
0	\downarrow $0 \xlongequal{\quad} \widetilde{HM}_{-4}(T^3) \xrightarrow{0} \widetilde{HM}_{-4}(Wh) \xlongequal{\quad} 0$	0

$$\begin{array}{cccccccccccc}
\bar{0} & \xrightarrow{i} & \check{0} & \xrightarrow{j} & \hat{0} & \xrightarrow{p} & \overline{-1} & \xrightarrow{i} & \check{-1} & \xrightarrow{j} & \widehat{-1} & \xrightarrow{p} & \overline{-2} & \xrightarrow{i} & \check{-2} & \xrightarrow{j} & \widehat{-2} & \xrightarrow{p} & \overline{-3} & \xrightarrow{i} & \check{-3} \\
\beta & \mathbb{Z} & \mathbb{Z} & 0 & 0 & \mathbb{Z} & \mathbb{Z}^2 & \mathbb{Z} & \mathbb{Z} & 0 & 0 & 0 \\
\oplus & \oplus \xrightarrow{\text{rk}=2} \oplus \xrightarrow{0} \oplus \xrightarrow{0} \oplus \xrightarrow{\text{rk}=2} \oplus \xrightarrow{\text{rk}=2} \oplus \xrightarrow{\text{rk}=1} \oplus \xrightarrow{\text{rk}=1} \oplus \xrightarrow{0} \oplus \xrightarrow{\text{rk}=2} \oplus \xrightarrow{0} \oplus \\
\alpha[1] & \mathbb{Z} & \mathbb{Z} & 0 & \mathbb{Z}^2 & \mathbb{Z}^3 & \mathbb{Z} & \mathbb{Z} & 0 & \mathbb{Z}^2 & \mathbb{Z}^2 & 0
\end{array}$$

Outside of this range, the homology is standard. (i.e. isomorphic to the homology of $2S^1 \times S^2$, which has the same first homology & triple intersection product as $Wh(2)$.) So $HM_{-1}(Wh(2), \mathfrak{s}_0) = \mathbb{Z}^2$ and $HM_*(Wh(2), \mathfrak{s}_n) = 0$ otherwise.

3 Zero surgery on the stevedore's Knot, 6_1

To obtain zero surgery on the stevedore's knot, we do surgery on a variant of the Borromean rings from Figure 2 where the orange (B) component is 0 framed, the green (C) and red (D) are -1 framed, and the blue component (A) is $+1$ framed. Call this 3-manifold St or Y_{+1} . Now we apply the $+1 \rightarrow \infty \rightarrow 0$ surgery exact sequence to the red curve to compute the Monopole Homology. Note that $Y_\infty \cong S^1 \times S^2$ and $Y_0 \cong Wh(2)$. We compute that $W_{\infty 0}$ induces a grading shift of -1 and that the map on \overline{HM} is induced from:

$$\begin{array}{ccc}
 S^1 \times S^2 & & Wh(2) \\
 \text{odd} & & \text{even} \\
 & & a \wedge b \\
 \text{pt} & \longrightarrow & \text{pt} \\
 \text{odd} & & \text{odd} \\
 & & a \\
 b & \longrightarrow & b
 \end{array}$$

α		β
Kernel		Cokernel
0	$0 \longleftarrow \widehat{HM}_2(S^1 \times S^2) \xrightarrow{0} \widehat{HM}_1(Wh(2)) \longrightarrow 0$	0
0	$\downarrow \mathbb{Z} \longleftarrow \overline{HM}_1(S^1 \times S^2) \xrightarrow{rk=1} \overline{HM}_0(Wh(2)) \longrightarrow \mathbb{Z}^2$	\mathbb{Z}
0	$\downarrow \mathbb{Z} \longleftarrow \widetilde{HM}_1(S^1 \times S^2) \xrightarrow{rk=1} \widetilde{HM}_0(Wh(2)) \longrightarrow \mathbb{Z}^2$	\mathbb{Z}
0	$\downarrow 0 \longleftarrow \widehat{HM}_1(S^1 \times S^2) \xrightarrow{0} \widehat{HM}_0(Wh(2)) \longrightarrow 0$	0
0	$\downarrow \mathbb{Z} \longleftarrow \overline{HM}_0(S^1 \times S^2) \xrightarrow{rk=1} \overline{HM}_{-1}(Wh(2)) \longrightarrow \mathbb{Z}^2$	\mathbb{Z}
0	$\downarrow \mathbb{Z} \longleftarrow \widetilde{HM}_0(S^1 \times S^2) \xrightarrow{rk=1} \widetilde{HM}_{-1}(Wh(2)) \longrightarrow \mathbb{Z}^4$	\mathbb{Z}^3
0	$\downarrow 0 \longleftarrow \widehat{HM}_0(S^1 \times S^2) \xrightarrow{0} \widehat{HM}_{-1}(Wh(2)) \longrightarrow \mathbb{Z}^3$	\mathbb{Z}^3
0	$\downarrow \mathbb{Z} \longleftarrow \overline{HM}_{-1}(S^1 \times S^2) \xrightarrow{rk=1} \overline{HM}_{-2}(Wh(2)) \longrightarrow \mathbb{Z}^2$	\mathbb{Z}
\mathbb{Z}	$\downarrow \mathbb{Z} \longleftarrow \widetilde{HM}_{-1}(S^1 \times S^2) \xrightarrow{0} \widetilde{HM}_{-2}(Wh(2)) \longrightarrow \mathbb{Z}$	\mathbb{Z}
0	$\downarrow \mathbb{Z} \longleftarrow \widehat{HM}_{-1}(S^1 \times S^2) \xrightarrow{rk=1} \widehat{HM}_{-2}(Wh(2)) \longrightarrow \mathbb{Z}^2$	\mathbb{Z}
0	$\downarrow \mathbb{Z} \longleftarrow \overline{HM}_{-2}(S^1 \times S^2) \xrightarrow{rk=1} \overline{HM}_{-3}(Wh(2)) \longrightarrow \mathbb{Z}^2$	\mathbb{Z}
0	$\downarrow 0 \longleftarrow \widetilde{HM}_{-2}(S^1 \times S^2) \xrightarrow{0} \widetilde{HM}_{-3}(Wh(2)) \longrightarrow 0$	0
0	$\downarrow \mathbb{Z} \longleftarrow \widehat{HM}_{-2}(S^1 \times S^2) \xrightarrow{rk=1} \widehat{HM}_{-3}(Wh(2)) \longrightarrow \mathbb{Z}^2$	\mathbb{Z}
0	$\downarrow \mathbb{Z} \longleftarrow \overline{HM}_{-3}(S^1 \times S^2) \xrightarrow{rk=1} \overline{HM}_{-4}(Wh(2)) \longrightarrow \mathbb{Z}^2$	\mathbb{Z}
0	$\downarrow 0 \longleftarrow \widetilde{HM}_{-3}(S^1 \times S^2) \xrightarrow{0} \widetilde{HM}_{-4}(Wh(2)) \longrightarrow 0$	0

	$\bar{0} \xrightarrow{i} \check{0} \xrightarrow{j} \hat{0} \xrightarrow{p} \overline{-1} \xrightarrow{i} \check{-1} \xrightarrow{j} \widehat{-1} \xrightarrow{p} \overline{-2} \xrightarrow{i} \check{-2} \xrightarrow{j} \widehat{-2} \xrightarrow{p} \overline{-3} \xrightarrow{i} \check{-3}$
β	$\mathbb{Z} \quad \mathbb{Z} \quad 0 \quad \mathbb{Z} \quad \mathbb{Z}^3 \quad \mathbb{Z}^3 \quad \mathbb{Z} \quad \mathbb{Z} \quad \mathbb{Z} \quad \mathbb{Z} \quad 0$
\oplus	$\oplus \xrightarrow{rk=1} \oplus \xrightarrow{0} \oplus \xrightarrow{0} \oplus \xrightarrow{rk=1} \oplus \xrightarrow{rk=3} \oplus \xrightarrow{0} \oplus \xrightarrow{rk=1} \oplus \xrightarrow{0} \oplus \xrightarrow{rk=1} \oplus \xrightarrow{0} \oplus$
$\alpha[1]$	$0 \quad 0 \quad 0 \quad 0 \quad \mathbb{Z} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0$

Outside of this range, the homology is standard. (i.e. isomorphic to the homology of $S^1 \times S^2$, which has the same first homology & triple intersection product as St .) So $HM_{-1}(St, \mathfrak{s}_0) = \mathbb{Z}^3$ and $HM_*(St, \mathfrak{s}_n) = 0$ otherwise.

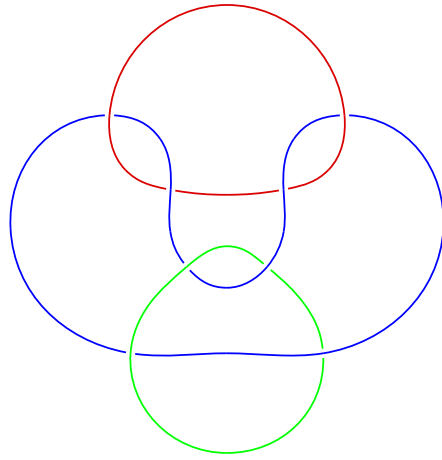


Figure 1: Borromean rings

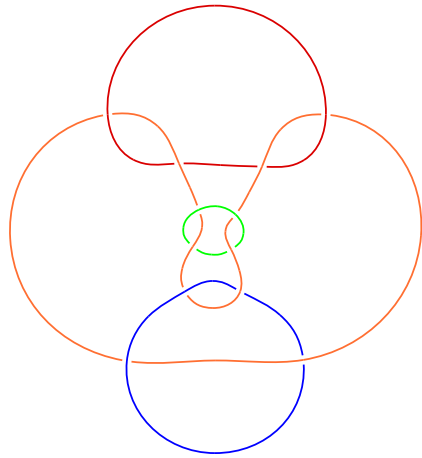


Figure 2: Modified Borromean rings