MATH W4052 PROBLEM SET 8
DUE APRIL 4, 2011.

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(1) Lickorish, Exercise 6.3.
(2) Let $M$ be a finitely presented module over a ring $R$. Prove that the $r^{th}$ elementary ideal of $M$, $E_r$, is independent of the choice of presentation matrix used to define it.
(3) Let $\Gamma_1$ and $\Gamma_2$ be embedded graphs in $\mathbb{R}^3$. Suppose that there is a ball $B$ in $\mathbb{R}^3$ so that $\Gamma_1 \cap (\mathbb{R}^3 \setminus B) = \Gamma_2 \cap (\mathbb{R}^3 \setminus B)$, and that $\Gamma_1 \cap B$ and $\Gamma_2 \cap B$ consist of two arcs each, and look like this (in the style of knot diagrams):

Prove: for any such $\Gamma_1, \Gamma_2$, $H_1(\mathbb{R}^3 \setminus \Gamma_1) \cong H_1(\mathbb{R}^3 \setminus \Gamma_2)$. Is the same true with $H_1$ replaced by $\pi_1$?
(4) In class, we considered the following covering space of the figure 8:

What subgroup of $\pi_1$ of the figure 8 does this correspond to? Show that this subgroup is not normal. (This may take some work.)

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