1. **Correlation**

Read the Correlation handout (posted on the webpage). Then:

1. Compute, by hand, the correlation between the vectors \((1, 3)\) and \((3, 1)\).
2. Compute, by hand, the correlation between the vectors \((1, 2, 3)\) and \((5, 2, 2)\).
3. Here are two situations when it is natural to compute the correlation of \(\vec{v}\) and \(\vec{w}\):
   
   (a) \(\vec{v}\) is the vector of scores on homework 1, and \(\vec{w}\) is the vector of scores on the final exam. So, if there are a hundred students in this class, \(\vec{v}\) and \(\vec{w}\) are both vectors in \(\mathbb{R}^{100}\).
   
   (b) \(\vec{v}\) is the vector of closing prices of Apple Inc.’s stock in 2014, and \(\vec{w}\) is the vector of temperatures, in degrees Fahrenheit, in central park at 9 a.m., on the days the market was open in 2014. (There were 252 trading days in 2014, so each of \(\vec{v}\) and \(\vec{w}\) is a vector in \(\mathbb{R}^{252}\).)

Here are two situations when it is *not* natural to compute the correlation of \(\vec{v}\) and \(\vec{w}\):

(c) Suppose our class has 10 homework assignments out of 100 points each, two midterms out of 100 points, and a final out of 100 points. Let \(\vec{v}\) be the vector of Alice’s scores, and let \(\vec{w}\) be the vector of Bob’s scores. So, \(\vec{v}\) and \(\vec{w}\) are vectors in \(\mathbb{R}^{13}\).

(d) Let \(\vec{v}\) be the vector \((\text{price of a barrel of oil in dollars, price of Apple stock in dollars, temperature in Central park in Fahrenheit, distance to the moon in feet})\) at 9:30 a.m. on Tuesday, and \(\vec{w}\) the same quantities but at 9:30 a.m. on Wednesday.

What is the difference? Why is correlation probably not a useful notion in cases (c) and (d)?
2. Mathematica

Download and install Mathematica. Create a new “notebook”. Then:

(1) Use Mathematica to compute 1+1: type 1+1 and hit shift-Return. You should see the numeral 2 in the next line.

(2) Use Mathematica’s “Print” function do display your name. For me, the command is \texttt{Print["Robert Lipshitz"]} followed by shift-Return. You should see your name printed in the next line.

(3) Define a new vector \( \mathbf{v} = \langle 1, 2, 3 \rangle \) by the command \texttt{v = {1, 2, 3}}. (Mathematica uses set braces to denote lists or vectors.)

(4) Define a new vector \( \mathbf{w} = \langle -1, 0, 1 \rangle \).

(5) The dot product is computed with a period. Do it: \texttt{v.w}.

(6) Length is computed with the norm function: \texttt{Norm[v]}. (Do it.)

(7) Aside: What does the length function do? i.e., what is \texttt{Length[v]}?

(8) Use Mathematica to compute the cosine of the angle between \( \mathbf{v} \) and \( \mathbf{w} \).

(9) You can compute arccos by \texttt{ArcCos}. Use this to compute the angle between \( \mathbf{v} \) and \( \mathbf{w} \).

(10) The answer to the previous step was rather unsatisfying. You can tell Mathematica you want a decimal (float), not a symbolic expression, by multiplying by 1.0: \texttt{1.0*ArcCos[..]} (with the dots replaced by the answer to step 8).

(11) A convenient shorthand: Mathematica keeps track of the answers to all previous computations. So if the answer to step 9 read: \texttt{Out[9]=ArcCos[..]} then you can write \texttt{1.0*Out[13]} to do step 10. (Try it.)

(12) Use Mathematica to check your answers to the first two correlation problems above, using vector arithmetic.

(13) There is also a Correlation function, \texttt{Correlation[v,w]}. Use it to check your work above.

(14) Print out your Mathematica worksheet (after deleting any junk commands that didn’t work) and turn it in as part of your problem set.

Please make a note of how long this Mathematica stuff took you, and whether you needed help: I think it should be easy, but I could be wrong. If you needed help, please note where, or what confused you.

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