This is version 2, with a correction to Problem (IV.1).

1. Some Stewart problems

(I.1) Stewart 14.4.6.
(I.2) Stewart 14.4.19.
(I.3) Stewart 14.4.46.
(I.4) Stewart 14.5.28, but do not use Equation 6: just apply the chain rule to the equation 
\[ \cos(xy) - 1 - \sin(y) = 0. \] (You may use equation 6 to check your answer, if you like.)
(I.5) Stewart 14.5.34, but do not use Equation 7: just apply the chain rule to the equation 
\[ yz + x\ln(y) - z^2 = 0. \] (You may use equation 7 to check your answer, if you like.)
(I.6) Stewart 14.5.42.
(I.7) Stewart 14.5.51.

2. Chain rule and change of coordinate systems

(II.1) Consider the parametric curve in polar coordinates \((r, \theta) = (1 + \sin(t), t)\).
   (a) Use the chain rule to find an expression for the speed of the curve at time \(t\).
   (b) Find the arc length of the curve between \(t = 0\) and \(t = 2\pi\).

(II.2) Consider a parametric curve in spherical coordinates \(\vec{r} = (\rho(t), \theta(t), \phi(t))\). Use the chain
rule to find an expression for the speed of the curve \(\vec{r}\). (Hint: imitate what we did in class for polar coordinates.)

3. Implicit differentiation

(III.1) Suppose \(x\) and \(y\) satisfy the relationship \(e^{x^2}y^3 = e^{xy}\). Use implicit differentiation to compute
\(y'(x)\) and \(x'(y)\) at \((1, 1)\).

(III.2) Suppose \(x, y,\) and \(z\) satisfy \(\sin(xyz) = xy + yz\). Use implicit differentiation to compute
\(\frac{\partial z}{\partial x}(1, 1, \pi)\) and \(\frac{\partial z}{\partial y}(1, 1, \pi)\).

(III.3) Consider the curve \(C\) defined by \(\cos(xy) - (x + y)/(2\sqrt{\pi}) = 0\) in \(\mathbb{R}^2\).
   • Notice that \((-\sqrt{\pi}, -\sqrt{\pi})\) lies on the curve. Using problem (IV.1), explain why the
curve \(C\) implicitly defines \(y\) as a function of \(x\) near \((-\sqrt{\pi}, -\sqrt{\pi})\).
   • Compute \(y'(x)\) at the point \((-\sqrt{\pi}, -\sqrt{\pi})\).
   • The point \((2\sqrt{\pi}, 0)\) also lies on the curve \(C\). Does \(C\) define \(y\) implicitly as a function of \(x\) near \((2\sqrt{\pi}, 0)\)? Does \(C\) define \(x\) implicitly as a function of \(y\) near \((2\sqrt{\pi}, 0)\)? Explain.
   • What happens when you try to compute \(y'(x)\) at \((2\sqrt{\pi}, 0)\) using implicit differentiation?

(III.4) Consider the ellipsoid \(E\) given by \(x^2/4 + y^2/9 + z^2 = 3\).
   • At the point \((2, 3, 1)\), \(E\) defines \(z\) implicitly as a function of \(x\) and \(y\). Compute \(\frac{\partial z}{\partial x}\) and
\(\frac{\partial z}{\partial y}\).
   • At what points on \(E\) does \(E\) not define \(z\) implicitly as a function of \(x\) and \(y\)?
4. MATHEMATICA

(IV.1) (This goes along with problem (III.3).) Use ContourPlot to plot the curve \( \cos(xy) - \frac{x+y}{2\sqrt{\pi}} = 0 \) with plot region \(-6 \leq x \leq 6, -6 \leq y \leq 6\). Plot it again with plot region \(-2.25 \leq x \leq -1, -2.25 \leq y \leq -1\).

(IV.2) Suppose we wanted to define \( f(u, v) = u^2 + v^2 \). In Mathematica, you do this like:
\[
f[u_, v_] := u^2 + v^2
\]
(Try it.)

(IV.3) Then you can compute \( f(1, 2) \) just as you would expect:
\[
f[1, 2]
\]
(Try it.)

(IV.4) Now, we can chain functions together. To define \( u(t) = e^t \) use:
\[
u[t_] := \text{Exp}[t]
\]
(Try it.)

(IV.5) Define \( v(t) = \sin(t) \).

(IV.6) We can now compose functions: \( f(u(t), v(t)) \) is
\[
f[u[t], v[t]]
\]
(Try it.)

(IV.7) Derivatives of compositions work exactly as you would expect; for instance:
\[
D[f[u[t], v[t]], t]
\]
(Try it.)

E-mail address: lipshitz@math.columbia.edu