Sample Final
Math UN1101: Calculus I, Section 10
Instructor: Linh Truong
Fall 2017

Name: _____________________________________________
UNI: _____________________________________________

Instructions:

• Print your name in the space above.
• Show your reasoning and intermediate computations.
• You have 2 hours and 50 minutes.
• No notes, books, calculators or any other electronic devices are allowed.
• Write answers in the space provided. If you need extra space, use the backs of pages and clearly label your work.

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1. (10 points) Mark True or False (1 point each). No justification is needed.

(a) T [F] If $f(x)$ is increasing at $x = 3$, then $f''(3) > 0$.

(b) T [F] The function $f(x) = \tan(x)$ is continuous at $x = \pi$.

(c) T [F] The derivative of $f(x) = e^{ex}$ is $f'(x) = e^{ex}+x$.

(d) T [F] The derivative of $f(x) = 1/(1 + x^2)$ is $f'(x) = \arctan(x)$.

(e) T [F] $\int_{1/2}^{1} \ln x \ dx$ is positive.

(f) T [F] $\int_{a}^{b} f(x)g(x) \ dx = (\int_{a}^{b} f(x) \ dx) \cdot (\int_{a}^{b} g(x) \ dx)$

(g) T [F] If $\int_{0}^{2} f(x)dx = 2$ and $\int_{2}^{5} f(x)dx = 6$, then $\int_{0}^{5} f(x)dx = 8$.

(h) T [F] The function $f(x) = \int_{0}^{x} (1 - t)e^{t^3} \ dt$ is decreasing when $x > 1$.

(i) T [F] The fundamental theorem of calculus ensures that

$$\frac{d}{dx} \int_{2}^{3} f'(t) \ dt = f(3) - f(2).$$

(j) T [F] The substitution rule ensures that $\int f(u)du = \int f(x^2)dx$ if $u = x^2$.
2. (4 points) Compute \( \lim_{{x \to 4}} \frac{x - 4}{{x^2 - 9x + 20}} \).
3. (3 points) \( \lim_{x \to -\infty} \frac{x^5 + 5x + 2}{2x^4 - 3} \).
4. (4 points) Compute \( \lim_{x \to 0} \left( \frac{1}{e^x - 1} - \frac{1}{x} \right) \).
5. (4 points) Is the function $f(x)$ continuous at $x = 0$? Explain.

$$f(x) = \begin{cases} 
 x^4 \sin\left(\frac{1}{x}\right) & \text{if } x > 0 \\
 e^{-x} - 1 & \text{if } x \leq 0 
\end{cases}$$
6. (4 points) Find $f'(x)$ if

$$f(x) = (\cos x)(\ln(6x + 1)).$$
7. (5 points) Find the tangent line of \( y = \frac{e^x}{x^{1/3} + 1} \) at \( x = 1 \).
8. (5 points) Let \( f(x) = x - 2 \arctan x \). Find the absolute maximum and absolute minimum of \( f(x) \) on the interval \([0, 4]\). \textit{Hint: \arctan(4) is about 1.3.}
9. (4 points) A particle moves along the curve $3x^2 + y^2 = 13$. At the point $(2, 1)$, the $x$-coordinate is increasing at a rate of 5 in/sec; in other words $\frac{dx}{dt} = 5$ in/sec. Find the rate of change of the $y$-coordinate at the point $(2, 1)$. 
10. (4 points) Evaluate the limit:

\[
\lim_{n \to \infty} \sum_{i=1}^{n} 4e^{-2x_i} \Delta x,
\]

where \( x_i = 1 + i \Delta x \) and \( \Delta x = \frac{3}{n} \).
11. (4 points) $\int_0^1 \sqrt{1 + 7x} \, dx$. 
12. (4 points) \( \int_{-3}^{0} (1 + \sqrt{9 - x^2}) \, dx \). Hint: Interpret the integral as the area of a region.
13. (5 points) Find $f'(\sqrt{\pi})$ if $f(x) = \int_0^x \cos t \, dt$. 
14. (5 points) Find the indefinite integral $\int \sec^2 x \tan^4 x \, dx$. 
15. Consider the function

\[ f(x) = \int_{1}^{x} (t^2 - 4t - 5) \, dt \]

(a) (3 points) Find the intervals on which \( f(x) \) is increasing or decreasing.

(b) (2 points) Find the intervals on which \( f(x) \) is concave up or concave down.
16. A function $f(t)$ satisfies $f''(t) = t + 5$ and $f'(0) = 4$.

(a) (2 points) Find $f'(t)$.

(b) (3 points) Find $\int_0^8 f'(t)dt$. 
17. (5 points) Find the area of the region enclosed by the curves $4x + y^2 = 12$ and $x = y$. 