1. (a) Find the fifth roots of 32.
(b) Find \((1 - i)^{50}\).

2. Find and classify (i.e., local maximum, local minimum, saddle point, or inconclusive) all critical points of the function:
\[ f(x, y) = y^3 - \frac{3}{2} y^2 + \frac{3}{2} x^2 - 3xy + 5. \]

3. Suppose that the temperature \(T\) on the circular plate, \(x^2 + y^2 \leq 1\), is given by
\[ T(x, y) = x^2 - x + 2y^2 \]
Find the hottest and coldest spots on the plate.

4. Let \(f(x, y) = x^2y\)
(a) Use Lagrangian multipliers method to find the absolute minimum and maximum values of \(f\) subject to the constraint \(x^2 + 2y^2 = 6\).
(b) Determine the absolute maximum and minimum values of \(f\) on the region described by \(x^2 + 2y^2 \leq 6\).

5. Let \(f(x, y) = 2x^2 - 2xy + 4y\). Find the absolute maximum and absolute minimum values of the function \(f(x, y)\) on the triangular region with vertices \((0, 0)\), \((1, 0)\) and \((0, 1)\).

6. Find all points on the surface \(S\) with equation \(x^2 + y^2 - z^2 = 1\) at which the tangent plane to \(S\) is parallel to the plane \(2x + y - z = 4\).

7. Let \(f(x, y)\) be a function of two variables such that the following directional derivatives are known at \((0, 0)\)
\[ D\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) f(0, 0) = -\sqrt{2} \quad D\left(\frac{2}{\sqrt{5}}, \frac{-1}{\sqrt{5}}\right) f(0, 0) = \sqrt{5} \]
Compute the gradient of \(f(x, y)\) at \((0, 0)\).

8. (a) Let \(z = 1 + xy + 3xy^2\), and let \(x\) and \(y\) be functions of \(s\) and \(t\) with \(x(0, 0) = 1, y(0, 0) = 2\), \(\frac{\partial x}{\partial s}(0, 0) = 3, \frac{\partial y}{\partial s}(0, 0) = 4, \frac{\partial x}{\partial t}(0, 0) = 5,\) and \(\frac{\partial y}{\partial t}(0, 0) = 6\).
1. Use the chain rule to find \(\frac{\partial z}{\partial s}\) and \(\frac{\partial z}{\partial t}\) when \(s = 0\) and \(t = 0\).
2. Use linear approximation at \(s = 0, t = 0\) to approximate the value of \(z\) when \(s = 0.1\) and \(t = 0.2\). You do not need to simplify your answer.
(b) A differentiable function \(z = f(x, y)\) is implicitly determined by the equation
\[ xz + zy^2 + y^3 - 3 = 0 \]
Find \(\frac{\partial z}{\partial x}\) and \(\frac{\partial z}{\partial y}\).

9. Find the limit if it exists or show that the limit does not exist.
(a) \[ \lim_{(x,y) \to (0,0)} \frac{x^2y + y^3}{x^2 + 2y^2} \]
10. Suppose that a function \( f(x, y) \) has the first order partial derivatives
\[
\begin{align*}
    f_x(x, y) &= ye^x + 3y^2x^4 + 5, \\
    f_y(x, y) &= e^x + kyx^5 - 7
\end{align*}
\]
for a constant \( k \).

(a) If all the second order partial derivatives of \( f \) are continuous for any \( x \) and \( y \), then what is the value of \( k \)?

(b) Using the value of \( k \) found in (a), compute the value of \( f_{xx} + f_{yy} \) when \( x = 1 \) and \( y = 1 \).

11. Consider the following two variable function
\[
f(x, y) = e^{\ln(4 - x^2 - 4y^2)}.
\]

(a) Find the domain of the function \( f \), and sketch the domain.

(b) Find the range of the function \( f \).