HTT refers to `https://arxiv.org/abs/math/0608040v4`.

All categories are ∞-categories. When we mean a 1-category we will explicitly say so.

Whenever we say "regular cardinal", you can just think of "cardinality".

Definition 1 (HTT 5.3.3.3)**.** Let κ be a regular cardinal. If J is a category, then J is κ-filtered if and only if J-indexed colimits of spaces commute with κ-small limits, that is, the colimit functor $\mathrm{Fun}(J, \mathcal{S}) \to \mathcal{S}$ preserves κ-small limits.

For another definition of filteredness, see HTT 5.3.1.7. (Also maybe read some of the following discussion).

Definition 2 (HTT 5.3.4.5)**.** Say a functor is κ-continuous if it preserves κ-filtered colimits. Say an object is compact if the functor it corepresents is κ-continuous.

See examples HTT 5.3.4.(1-3) for 1-categorical examples of compactness.

Following the reference in this definition allows us to think of Ind_κ as freely adjoining κ-filtered colimits.

Definition 3 (HTT 5.3.5.4 (and 5.3.5.1))**.** Let $\mathcal{C}$ be a small category. Then $\mathrm{Ind}_\kappa(\mathcal{C}) \subseteq \mathrm{Psh}(\mathcal{C})$ is the full subcategory of presheaves which preserve κ-small limits

See page 341 of HTT (beginning of 5.4) for motivating discussion.

Definition 4 (HTT 5.4.2.1, 5.4.2.2)**.** A κ-accessible category is a category which is Ind_κ of a small category. Equivalently, it is generated under κ-filtered colimits by an essentially small full subcategory of κ-compact objects.

HTT 5.4.2.3, 5.4.2.4 are cool.

Definition 5 (HTT 5.4.2.1, 5.4.2.5)**.** An accessible category is one which is κ-accessible for some κ. An accessible functor is one which is κ-continuous for some κ.

Definition 6 (HTT 5.5.0.18 (5.5.0.1 in newer versions), 5.5.1.1)**.** A category is presentable if it is accessible and has all small colimits. Equivalently, a category is presentable if it is an accessible localization[1] of a presheaf category[2].

Proposition 7 (HTT 5.5.2.4)**.** *Presentable categories admit small limits.*

Theorem 8 (Adjoint functor theorem HTT 5.5.2.9)**.** *A functor between presentable categories*

- *admits a right adjoint if and only if it preserves small colimits.*
- *admits a left adjoint if and only if it is accessible and preserves small limits.*

From 5.5.2.9 we can deduce the results 5.5.2.2, 5.5.2.7 which were used to prove it, and which give criteria for the (co)representability of (co)presheaves.

Definition 9 (HTT 5.5.3.1)**.** Define $\mathbf{Pr^L}$ to be the category of presentable categories with colimit-preserving functors between them.

Theorem 10 (5.5.3.13, 5.5.3.18)**.** *Limits in* $\mathbf{Pr^L}$ *exist and are computed as colimits of categories. Colimits in* $\mathbf{Pr^L}$ *exist and are computed by first taking right adjoints and then taking the limit of categories.*

Other nice things to know: 5.3.5.3, 5.3.5.5, 5.3.5.10, 5.3.5.12, 5.3.5.13, 5.5.1.9

[1] This means that it is a reflective subcategory where the reflector (the localization functor) is accessible.

[2] This means $\mathrm{Psh}(\mathcal{C})$ for $\mathcal{C}$ small.