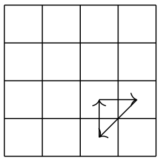
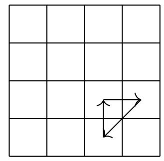
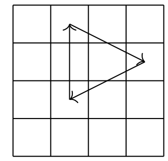
HOW TO SOLVE THE 15-PUZZLE BLINDFOLDED: SOLUTIONS TO EXERCISES

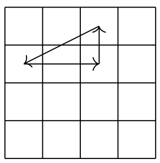
1. 3-cycles

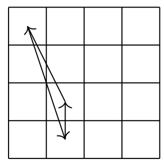
Problem 1: Understanding 3-cycle notation.

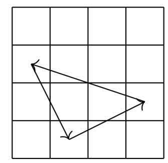
(11 12 15) =  ,

(12 15 11) =  ,

(10 2 8) =  ,

(3 5 7) =  ,

(1 14 10) =  ,

(5 14 12) =  .

Problem 2: Applying 3-cycles.

1	5	8	10
11	2	6	9
14	12	3	7
4	15	13	

(11 12 15) =

1	5	8	10
11	2	6	9
14	12	13	3
4	15	7	

,

1	5	8	10
11	2	6	9
14	12	3	7
4	15	13	

(10 2 8) =

1	12	8	10
11	2	6	5
14	9	3	7
4	15	13	

,

7	12	11	14
10	3	15	13
2	6	9	8
5	4	1	

(3 5 7) =

7	12	15	14
11	3	10	13
2	6	9	8
5	4	1	

,

7	12	11	14
10	3	15	13
2	6	9	8
5	4	1	

(1 14 10) =

6	12	11	14
10	3	15	13
2	4	9	8
5	7	1	

.

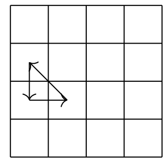
Problem 3: Identifying 3-cycle needed to solve a state.

1. (11 12 15) 2. (1 8 10) 3. (2 8 12) 4. (7 13 14)

Date: April 22, 2017.

2. Basic 3-cycles

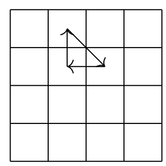
1. Fill in the blanks to explain how to perform the 3-cycle

(5 9 10) = 

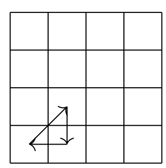
in three steps:

Step 1: Make position 6 blank by doing the following moves: D2R2
Step 2: Use this blank to perform the 3-cycle in 4 moves: URDL
Step 3: Reverse the moves from Step 1: L2U2

3.

(2 7 6) = 

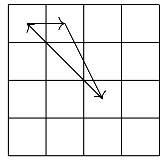
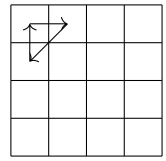
Step 1: Make position 3 blank by doing the following moves: D3R
Step 2: Use this blank to perform the 3-cycle in 4 moves: RULD
Step 3: Reverse the moves from Step 1: LU3

(14 13 10) = 

Step 1: Make position 9 blank by doing the following moves: D3R3U2
Step 2: Use this blank to perform the 3-cycle in 4 moves: ULDR
Step 3: Reverse the moves from Step 1: D2L3U3

3. Easy 3-cycles

1. Consider the following 3-cycles:

$(1\ 2\ 11) =$  , $(1\ 2\ 5) =$  .

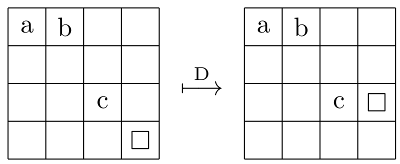
We want to reduce the 3-cycle (1 2 11) to the basic 3-cycle (1 2 5) using conjugation:

$$(1\ 2\ 11) = x(1\ 2\ 5)x^{-1}$$

for some sequence of moves x. For this to work, x should:

- move the tile in position _11_ to position _5_;
- make position _6_ blank;
- without affecting positions _1_ and _2_.

Here's one way to find such an x. First, apply D so that position 12 is now blank:



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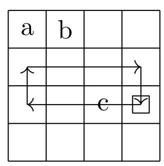
The point of doing this is that positions 5, 8, 12 now lie on a loop:



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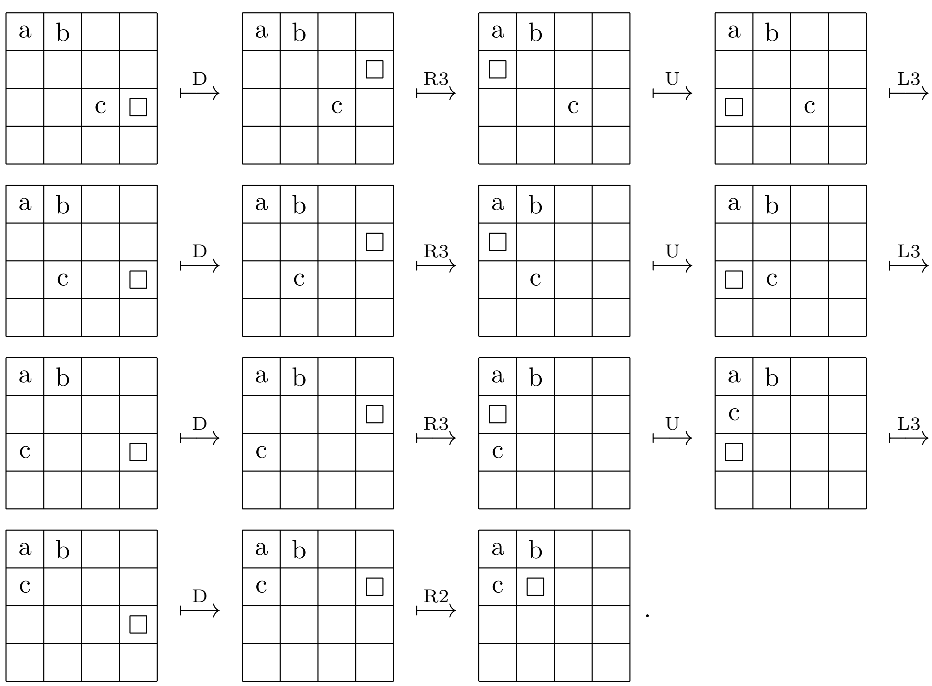
Now, we can use this loop to do what we want. That is, we can take

$$x = \text{D(DR3UL3)(DR3UL3)}.$$

Here's how this looks step by step, starting with the second D:



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4. Hard 3-cycles

1. There are several reasonable choices of d. Here's one:

$$(2\ 8\ 10) = (2\ 8\ 3)(2\ 3\ 10).$$

2. Reduce each easy 3-cycle to a basic 3-cycle using conjugation:

$$(2\ 8\ 3) = x(2\ 7\ 3)x^{-1}, \quad \text{where} \quad x = (\text{RD2LU2})\text{R2D2},$$
$$(2\ 3\ 10) = y(2\ 3\ 6)y^{-1}, \quad \text{where} \quad y = (\text{D2R2U2L2})\text{D2R}.$$

5. Cycle decomposition and simplification

Problem 1: Cycle decomposition.

1. (1 7 13 5 4 14 2 11 9 3 15 6 12 10 8)
2. (1 10 5 2 14 15 7)(3 8 6)(4 13 11)(9)(12)
3. (1 5 12 2 15 11 8 14 10 3 4)(6)(7 13 9)
4. (1 4 3)(2 12 8 14)(5 10 7 13 9 6 15 11)
5. (1 12 11 8 13 15 5 14 4 7)(2 10 9 3)(6)
6. (1 7 10 9 5 12)(2 3 11 13 8 14)(4 15 6)
7. (1)(2 7 6 8 12 15)(3 14 4 13 11)(5 10)(9)
8. (1 15)(2 14)(3 13)(4 12)(5 11)(6 10)(7)(8)(9)

Problem 2: Cycle simplification.

1. (1 7 13)(1 5 4)(1 14 2)(1 11 9)(1 3 15)(1 6 12)(1 10 8)
2. (1 10 5)(1 2 14)(1 15 7)(3 8 6)(4 13 11)
3. (1 5 12)(1 2 15)(1 11 8)(1 14 10)(1 3 4)(7 13 9)
4. (1 4 3)(2 12 8)(2 14)(5 10 7)(5 13 9)(5 6 15)(5 11)
5. (1 12 11)(1 8 13)(1 15 5)(1 14 4)(1 7)(2 10 9)(2 3)(6)
6. (1 7 10)(1 9 5)(1 12)(2 3 11)(2 13 8)(2 14)(4 15 6)
7. (2 7 6)(2 8 12)(2 15)(3 14 4)(3 13 11)(5 10)(9)
8. (1 15)(2 14)(3 13)(4 12)(5 11)(6 10)(7)(8)(9)

Problem 3: Pairs of 2-cycles.

1. no 2-cycle left
2. no 2-cycle left
3. no 2-cycle left
4. (2 14)(5 11) = (2 14 5)(2 11 5)
5. (1 7)(2 3) = (1 7 2)(1 3 2)
6. (1 12)(2 14) = (1 12 2)(1 14 2)
7. (2 15)(5 10) = (2 15 5)(2 10 5)
8. (1 15)(2 14)(3 13)(4 12)(5 11)(6 10) = (1 15 2)(1 14 2)(3 13 4)(3 12 4)(5 11 6)(5 10 6)