Name: _____

Calculus III Practice Midterm 1

Instructor: Maksym Fedorchuk

No calculators will be allowed on the exam.

1. (*5 points*) If $\vec{v} \times \vec{w} = \langle 1, 1, -1 \rangle$ and $\vec{v} \cdot \vec{w} = 2$, find the angle between $\vec{v}$ and $\vec{w}$.

2. (*10 points*) For which values of parameter a, the vectors $\vec{u} = \langle 1, a, 2 \rangle$ and $\vec{v} = \langle a, 4, 4 \rangle$ are

(a) parallel?

(b) orthogonal?

(c) make 45° angle?

3. (*15 points*) Find an equation of the following lines (if they exist!):

a. (*5 pts*) The line through $A = (2, 4, 2), B = (3, 7, -2), C = (1, 3, 3)$.

b. (*5 pts*) The line that passes through the points and $(3, 7, 0)$ and $(-5, 5, 1)$.

c. (*5 pts*) The line through the point $(0, 1, 1)$ and perpendicular to the vector $\langle 4, 6, -2 \rangle$.

4. (*10 points*) Reparametrize the curve $\vec{r}(t) = \langle e^t, e^t \sin t, e^t \cos t \rangle$ with respect to arc length measured from the point $(1, 0, 1)$.

5. (*10 points*) For the curve

$$\vec{r}(t) = \langle \frac{1}{3}t^3, \frac{1}{2}t^2, t \rangle,$$

find the unit normal and binormal vectors at every point. What is the equation of the osculating plane at $(0, 0, 0)$ and $(1/3, 1/2, 1)$?

6. (*10 points*) An object moves along a parabola in the plane with the position vector given by $\vec{r}(t) = \langle t, t^2 \rangle$. At what point in time is the normal component of the acceleration attains the maximal value? What is the curvature at that point?

7. (*10 points*) Convert the polar equation $r = \sin \phi$ from polar coodinates to cartesian coordinates and identify the curve it describes.

8. (*15 points*)

a. (*10 pts*) Find the plane perpendicular to the planes $x + y - z = 1$ and $2x - 3y + 4z = 5$ and passing through the point $P = (1, 0, -2)$.

b. (*5 pts*) What is the distance from P to the plane $2x - 3y + 4z = 5$?