Calculus III, Practice Midterm

Instructor: Maksym Fedorchuk

1. (*15 points*) Consider the quadric surface in 3-space defined by the equation

$$-x^2 - y^2 + \frac{z^2}{4} = 1$$

a. (*5 pts*) What are the traces for $z = 4$? For $z = 0$? For $z = 2$? For the three cases, you should both write down the equation for the trace and describe the set of solutions.

b. (*5 pts*) Describe the intersection of the surface and the tangent plane to the surface at the point $(0, 0, 2)$. What is the projection of the said intersection to the (x, y)-plane?

c. (*5 pts*) Find all points on this quadric surface where the tangent plane is parallel to $x + y + z = 0$.

2. (*15 points*)

a. (*5 pts*) Does the following limit exist?

$$\lim_{(x,y)\to(0,0)} \frac{xy^3+y^6}{x^2+y^6}$$

b. (*5 pts*) Compute the following limit:

$$\lim_{(x,y)\to(1,1)} \frac{\sin(x^2+y^2-2)}{(x^2+y^2-2)}.$$

c. (*5 pts*) Show that the function

$$f(x,y) = \begin{cases} \frac{x^2y+xy^2}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

is continuous at all points.

3. (*15 points*)

a. (*8 pts*) Let $f(x, y)$ be a function in two variables with continuous second partial derivatives. Define $g(t) = f(t^2, t^3)$. Express $g'(t)$ in terms of the partial derivatives of f.

b. (*7 pts*) With $g(t)$ defined as above, express $g''(t)$ in terms of the first and second order partial derivatives of f.

4. (*10 points*) Let $f(x,y) = x^2y^2 - x^2 - y^2$.

a. (*5 pts*) Find all points points such that

$$f(x,y) = \frac{\partial f}{\partial x}(x,y) = \frac{\partial f}{\partial y}(x,y) = 0.$$

(What can you say about the tangent plane at the above points?)

b. (*5 pts*) Find the partial derivatives $\dfrac{\partial f(r\cos t, r\sin t)}{\partial t}$ and $\dfrac{\partial f(r\cos t, r\sin t)}{\partial r}$