Advanced Exam Syllabus

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1 Symplectic and Contact Geometry

References: Geiges *Intro. to Contact Geometry.* McDuff & Salamon *Intro. to Symplectic Topology.*
DeRham cohomology: Sketch proofs of DeRham's theorem and Poincaré lemma.
Symplectic manifolds: provide examples including T^*M, compute symplectic area.
Contact manifolds: provide examples, discuss symplectizations, Boothby-Wang bundles, and other interplays between contact and symplectic manifolds.
Moser method: prove symplectic structures are stable using method, state applications of the method.
Discuss c_1 of symplectic vector bundles, relation to curvature integrals of line bundles over surfaces.

2 Morse Theory

References: Audin & Damian *Morse Theory and Floer Homology.*
Sketch proof: Morse functions are abundant and critical points are isolated, discuss Hessian.
Describe the Morse complex, explain $\partial^2 = 0$, compute H_{Morse} for S^n, $\mathbb{C}P^n$, $\mathbb{R}P^n$, $\mathbb{T}^n$.
Discuss isomorphism with cellular homology.

3 Hamiltonian Floer Homology and J-Holomorphic Curves

References: McDuff & Salamon *Introduction to Symplectic Topology.* Robbin & Salamon *Maslov Index for Paths of Symplectic Matrices (1992).* Salamon *Lectures on Floer Homology.*
State the Arnold conjecture and provide a brief history of it.
Symplectic action: know definitions of the one form Ψ_H and the smooth circle valued α_H on $\mathcal{L}M$. Prove that $d\alpha_H = \Psi_H$ and that the critical set of α_H is the set of 1-periodic Hamiltonian trajectories.
State the Hamiltonian-Floer PDE, prove that this defines the negative gradient flow of α_H on $\mathcal{L}M$.
Know the definitions of: monotone symplectic manifolds, symplectically aspherical manifolds. Explain monotonicity's importance in bubbling issues.
Provide examples and nonexamples of monotone symplectic manifolds.
State the definition of the Conley-Zehnder index of a path of symplectic matrices using the crossing form.
State properties of the Conley-Zehnder index and compute μ_{CZ} for some paths in $\text{Sp}(2)$.
Discuss the Floer chain complex, invariance of homology, and isomorphism with Morse homology.
State properties of moduli spaces of closed J-holomorphic curves.
Sketch proof of the dimension formula for moduli spaces of simple closed curves.
State the Riemann-Roch and Sard-Smale theorems.