1. For every irrep of $V$ of $S_3$, use Frobenius reciprocity and the character table of $S_4$ to decompose $\text{Ind}_{S_3}^{S_4} V$ into irreps.

2. What are all double cosets $S_{n-1}\backslash S_n/S_{n-1}$?

3. Let $Q$ be the 8 element quaternion group, let $H_i$ be the subgroup generated by $i$ and $H_j$ be the subgroup generated by $j$. For each irrep $W$ of $H_i$, compute $\text{Res}_H^Q \text{Ind}_H^Q W$.

4. For which representations $W$ of $A_4$ is $\text{Ind}_{A_4}^{S_4} W$ irreducible?

5. Let $\text{Cl}(G)$ denote the vector space of class functions on $G$. Let $\{H_i\}_{i \in I}$ be the set of cyclic subgroups of $G$. Show that the restriction map $\text{Res}: \text{Cl}(G) \to \bigoplus \text{Cl}(H_i)$ is injective.

6. With the same notation as above, show that the induction map $\text{Ind}: \bigoplus \text{Cl}(H_i) \to \text{Cl}(G)$ (which sends $(f_i)_{i \in I} \mapsto \sum_{i \in I} \text{Ind}_{H_i}^G f_i$) is surjective.

7. We call a representation of $G$ monomial if it can be written as an induction of a 1-dimensional representation of a subgroup of $H$, and we call a character monomial if it is the character of a monomial representation. Prove that every character of $G$ can be written as a $\mathbb{C}$-linear combination of monomial characters.

8. Show that every character can be written as a $\mathbb{Q}$-linear combination of monomial characters. (This is a theorem of Artin’s which is important in number theory. An even more important theorem says you can replace $\mathbb{Q}$ with $\mathbb{Z}$, but this is much much harder.)