

Possible Paper Topics.

1. The real case of the class number formula.

We'll be doing the imaginary case of this formula in class. I would definitely like someone to do this topic. Good sources include our textbook, and Dirichlet's *Lectures on Number Theory*.

2. Special values of the ζ -function.

The numbers $\zeta(2n)$ were found by Euler. There are several ways to go about computing them, and explaining a couple of these would make a nice paper. See the last chapter of Neukirch's "Algebraic Number Theory," for one way of doing it using methods from Riemann's first proof of the functional equation. I do not know of a particularly good source for making rigorous Euler's original argument involving an infinite product expansion of $\frac{\sin x}{x}$ but there must be several out there.

3. The functional equation for the Dirichlet L -series.

Just like the Riemann zeta function, the Dirichlet L -series also have an Analytic continuation and functional equation. See Neukirch's "Algebraic Number Theory" or our class textbook.

4. The prime number theorem in $\mathbb{Z}[i]$.

There is a remarkably similar version of the prime number theorem which holds in $\mathbb{Z}[i]$. The only treatment of this which I know of is in Hlwaka's "Analytic and Geometric Number Theory." However, one could probably work through a proof of this without a source.

5. A zero-free region for the ζ -function.

To get better bounds on the error in the Prime Number Theorem one needs to get better bounds on where the zeros of the ζ -function can lie. One such better bound is due to De La Valee Poussin and can be found in many sources, for example our textbook and Edwards's "Riemann's Zeta Function."

6. Newman's short proof of the Prime Number Theorem.

This is the fastest proof that I've seen of the prime number theorem. It can be found in Hlwaka "Analytic and Geometric Number Theory" as well as in an extremely brief article in the monthly. A good paper would be somewhat more detailed than that article, but a lot more concise than Hlwaka.

7. The Elementary (i.e. no complex analysis) Proof of the Prime Number Theorem.

From what I gather this is pretty painful, but some people seem to be interested in this sort of thing.

8. The Dedekind ζ -function and the general class number formula.

This paper requires a decent working knowledge of Algebraic Number Theory.

9. An explicit formula for $\psi(x)$.

Riemann's arguments suggested not only the prime number theorem, but an explicit formula for π in terms of the zeros of the ζ -function. For sources, see our textbook or Edwards's "Riemann's Zeta Function."

10. A paper of your choice