• **Answer** the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

• **NO calculators or other electronic devices, books or notes** are allowed in this exam.

• Please make sure the solutions you hand in are legible and lucid. You may only use techniques we have developed in class.

• You will have **30 minutes** to complete this quiz.

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1. True or False? No justification needed.

(a) **1 point** \( \mathbf{k} \times \mathbf{j} = \mathbf{i} \).

False. \( \mathbf{k} \times \mathbf{j} = -\mathbf{i} \)

(b) **1 point** The vector \( \langle -1, -2, -1 \rangle \) is normal to the plane \( x + 2y + z = 5 \).

True. The plane can be written \( -x - 2y - z = -5 \), from which it is clear that \( \langle -1, -2, -1 \rangle \) is a normal vector.

(c) **1 point** For any vector \( \mathbf{v} \), the cross product \( \mathbf{v} \times \mathbf{v} = \mathbf{0} \).

True. \( |\mathbf{v} \times \mathbf{v}| = |\mathbf{v}|^2 \sin \theta = 0 \), since \( \theta = 0 \).

(d) **1 point** The lines \( \mathbf{r}_1(t) = \langle 2, 3, 5 \rangle + t\langle -1, 1, 1 \rangle \) and \( \mathbf{r}_2(t) = \langle -4, 6, -9 \rangle + t\langle -2, 2, 2 \rangle \) are parallel.

True.

(e) **1 point** The curve of intersection of the cylinder \( y^2 + z^2 = 1 \) with the plane \( y + z = 10 \) is an ellipse.

False. This cylinder and this ellipse do not intersect. This can be seen by sketching the surfaces.
2. **5 points** Find an equation for the line of intersection of the planes $x + y + 2z = 1$ and $y + z = 1$.

*Solution:* Using the second equation, we solve for the variable $y$:

$$y = 1 - z.$$  

We can then substitute this expression into the first equation $x + y + 2z = 1$.

$$x + (1 - z) + 2z = 1.$$  

We can then solve for the variable $x$:

$$x = -z.$$  

We have therefore shown that any point $(x, y, z)$ on the line of intersection must be of the form $(-z, 1 - z, z)$. An equation for the line of intersection is

$$\mathbf{r}(t) = (-t, 1 - t, t).$$
3. (a) **4 points** At what points does the curve \( \mathbf{r}(t) = \langle t, t, 2t \rangle \) intersect the surface given by the equation \( x^2 + y^2 + z^2 = 12 \)?

**Solution:** We substitute the line into the equation of the sphere.

\[
t^2 + t^2 + (2t)^2 = 12.
\]

\[
6t^2 = 12.
\]

Solving for \( t \), we see that the times of intersection are \( t = \sqrt{2} \) and \( t = -\sqrt{2} \). Therefore the intersection points are

\[
(\sqrt{2}, \sqrt{2}, 2\sqrt{2}), (-\sqrt{2}, -\sqrt{2}, -2\sqrt{2}).
\]

(b) **1 point** Sketch the curve intersecting the surface.

![Diagram of a sphere with intersecting curves](image)
4. **5 points** Find an equation for the plane through the origin and the points $(1, 1, 1)$ and $(1, 2, 3)$.

*Solution:* We first find two vectors which lie on the plane:

\[ \mathbf{v}_1 = (1, 1, 1), \quad \mathbf{v}_2 = (1, 2, 3). \]

Take their cross product to find a vector normal to the plane.

\[ \mathbf{v}_1 \times \mathbf{v}_2 = (3 - 2, -(3 - 1), 2 - 1) = (1, -2, 1). \]

An equation for the plane is therefore

\[ x - 2y + z = d, \]

where $d$ is a constant to be determined. Since the origin $(0, 0, 0)$ lies on the plane, we must have

\[ 0 - 2(0) + 0 = d. \]

Therefore an equation for the plane is

\[ x - 2y + z = 0. \]