• **Answer** the questions **in the spaces provided** on the question sheets. If you run out of room for an answer, continue on the back of the page.

• **NO calculators or other electronic devices, books or notes** are allowed in this exam.

• Please make sure the solutions you hand in are **legible and lucid**. You may only use techniques we have developed in class.

• You will have **30 minutes** to complete this quiz.

<table>
<thead>
<tr>
<th>Question</th>
<th>Points</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td></td>
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<tr>
<td>2</td>
<td>5</td>
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<tr>
<td>Total:</td>
<td>20</td>
<td></td>
</tr>
</tbody>
</table>
1. True or False? No justification needed.
   (a) [1 point] For any function \( f(x,y) \) whose domain is \( \mathbb{R}^2 \), then
   \[
   \lim_{(x,y) \to (3,4)} f(x, y) = f(3, 4).
   \]
   Solution: False. This is only true if \( f \) is continuous.

   (b) [1 point] If \( f(x,y) \to L \) as \( (x,y) \to (a,b) \) along every straight line through \( (a,b) \), then
   \[
   \lim_{(x,y) \to (a,b)} f(x, y) = L.
   \]
   Solution: False. A counterexample is the limit of \( f(x,y) = \frac{x^2y}{x^4+y^2} \) as \( (x,y) \to (0,0) \). Straight line paths give a limit of zero, but the path \( (t,t^2) \) gives a limit of \( 1/2 \).

   (c) [1 point] If
   \[
   f(x, y, z) = z\sqrt{z^2 + y^2} + \frac{xy}{\sqrt{2 + xy}},
   \]
   then we have
   \[
   \frac{\partial}{\partial z} \frac{\partial}{\partial x} \frac{\partial f}{\partial y} = \frac{xy + xz + yz}{(\sqrt{z^2 + y^2})^2(\sqrt{2 + xy})}.
   \]
   Solution: False. Neither \( z\sqrt{z^2 + y^2} \) nor \( \frac{xy}{\sqrt{2 + xy}} \) contains all three variables \( x, y, z \), so the indicated third order derivative is zero.

   (d) [1 point]
   \[
   \frac{\partial}{\partial y} y^x = xy^{x-1},
   \]
   for \( y > 0 \).
   Solution: True.

   (e) [1 point] Let \( L(x, y) \) be the linearization of a function \( z = f(x, y) \) at a point \( (a, b) \). Then \( L(a, b) = f(a, b) \).
   Solution: True. By definition
   \[
   L(x, y) = f(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b).
   \]
   Hence \( L(a, b) = f(a, b) \).
2. 5 points Find the limit, if it exists, or show that the limit does not exist.

\[ \lim_{(x,y) \to (0,0)} \frac{x^2 y}{2x^3 + 5y^3}. \]

Solution: First try the path \( r(t) = \langle t, 0 \rangle \to \langle 0, 0 \rangle \) as \( t \to 0^+ \).

\[ f(t, 0) = 0. \]

Therefore

\[ \lim_{t \to 0^+} f(t, 0) = 0. \]

Next try the path \( r(t) = \langle t, t \rangle \to \langle 0, 0 \rangle \) as \( t \to 0^+ \).

\[ f(t, t) = \frac{t^3}{2t^3 + 5t^3} = \frac{1}{7}. \]

Therefore

\[ \lim_{t \to 0^+} f(t, t) = \frac{1}{7}. \]

It follows that

\[ \lim_{(x,y) \to (0,0)} \frac{x^2 y}{2x^3 + 5y^3} \]

does not exist.
3. (a) \[2 \text{ points}\] Let
\[f(x, y) = \frac{xy}{\sqrt{2x^2 + y^2}}.\]
Compute
\[\frac{\partial f}{\partial x}.\]

Solution: Compute using the product rule
\[
\frac{\partial f}{\partial x} = \frac{y}{\sqrt{2x^2 + y^2}} - \frac{xy}{2(2x^2 + y^2)^{3/2}}(4x)
\]
\[
= \frac{y(2x^2 + y^2) - 2x^2y}{(2x^2 + y^2)^{3/2}}
\]
\[
= \frac{y^3}{(2x^2 + y^2)^{3/2}}.
\]

(b) \[3 \text{ points}\] Let
\[f(x, y) = \sin(2xy)e^{-y}.\]
Compute \(f_{xy}\).

Solution: Take a first derivative
\[f_x = 2y\cos(2xy)e^{-y}.\]
Take a second derivative using the product rule
\[f_{xy} = 2\cos(2xy)e^{-y} - 4xy\sin(2xy)e^{-y} - 2y\cos(2xy)e^{-y}.\]
4. 5 points Find an equation for the tangent plane to the surface \( z = x \cos(x + y) \) at the point \((-1, 1, 1)\).

**Solution:** Rewrite in the form

\[ F(x, y, z) = x \cos(x + y) - z = 0. \]

We compute

\[ \nabla F = \langle F_x, F_y, F_z \rangle = (\cos(x + y) - x \sin(x + y), -x \sin(x + y), -1). \]

The normal to the plane at \((-1, 1, 1)\) is

\[ \langle \cos(-1 + 1) - (1) \sin(-1 + 1), -(1) \sin(-1 + 1), -1 \rangle = \langle 1, 0, -1 \rangle. \]

Therefore an equation for the plane is

\[ x - z = d, \]

where \(d\) can be determined by using the point \((-1, 1, 1)\).

\[ d = -1 - 1 = -2. \]

The equation of the tangent plane at \((-1, 1, 1)\) is therefore

\[ x - z = -2. \]