Brief Reminder of what we have done.

$S|k$ smooth

- $MIC(S|k) \longleftrightarrow QCoh(\mathcal{O}_S, D_S)$

- For any local section $\partial \in T_S$,

$\partial^{[p]} := \underbrace{\partial \circ \cdots \circ \partial}_{p}$ is a derivation

$\rightsquigarrow \partial^p - \partial^{[p]} \in D_S^{\leq p} \subseteq D_S$

$\rightsquigarrow$ if $(E, \nabla) \in MIC(S/k)$, then

$\partial^p - \partial^{[p]} \curvearrowright \{\text{sections of } E\}$

$\psi_\nabla(\partial) \in \underline{End}_k(E)$ is the "p-curvature"

$\rightsquigarrow T_S \longrightarrow \underline{End}_k(E)$

abelian sheaves

- Using $\{-,-\}$ + the fact that $D_S^{\leq p-1} \curvearrowright \mathcal{O}$ is faithful, proved that map (of abelian sheaves)

$$T_S \longrightarrow D_S \quad \text{of sheaves}$$

is p-linear

$\iff$ $T_{S'} \longrightarrow F_{S/k *} D_S$ is $\mathcal{O}_{S'}$-linear.

k perf

$$S = k[t] \Rightarrow S' = k[t]$$

$$T_{S'} \hookrightarrow k[t]\langle y \rangle$$

p-curv

$$k[t]\langle y\rangle \longrightarrow k\langle t, \partial\rangle$$

$$fy \longmapsto (f\partial)^p - (f\partial \circ \cdots \circ f\partial)$$

$$p\text{-linearity} \Rightarrow \quad y \longmapsto \partial^p - \partial^{[p]} = \partial^p$$

$$\Rightarrow fy \longmapsto f^p \partial^p$$

The p-curvature map induces:

$$\iota : T_{S'} \longrightarrow (F_{S/k})_* D_S$$

① $\mathrm{Im}(\iota) \subseteq Z(F_{S/k *} D_S)$

$\rightsquigarrow \tilde{\iota} : \mathrm{Sym}^* T_{S'} \xrightarrow{\sim} Z((F_{S/k})_* D_S)$

$\Rightarrow$ $(F_{S/k})_* D_S$ is a quasi-coherent sheaf over $T^*_{S'}$.

② $(F_{S/k})_* D_S$ is Azumaya / $T^*_{S'}$

(étale locally matrix algebra)

<u>Note</u>: The "action" of $\mathrm{Sym}^* T_{S'}$ on $(F_{S/k})_* D_S$ is induced from the p-curvature.

(2') Given

$$\begin{tikzcd}
(E,\nabla) \in MIC(S/k) \arrow[d, leftrightarrow] \\
E \in QCoh(\mathcal{O}_S, D_S) \arrow[d] \\
F_{S/k *} E \in QCoh(\mathcal{O}_{S'}, (F_{S/k})_* D_S) \arrow[d, "\text{Morita}"] \\
\text{Morita}(E,\nabla) \in QCoh(T^*_{S'})
\end{tikzcd}$$

(3) $(F_{S/k})_* D_S$ splits over the zero-section

$$S' \hookrightarrow T^*_{S'}$$

p-curvature acts by 0.