CALCULUS II ASSIGNMENT 11

DUE APRIL 25, 2019

1. Perhaps a somewhat enlightening way to solve second-order linear differential equations is to use some sort of series method. We will solve the equations

\[ y'' = y \quad \text{and} \quad y'' = -y \]

using this method.

(i) Assume that the equation \( y'' = y \) admits a solution \( f \) which is a function with a Taylor series expansion

\[ f(x) = \sum_{n=0}^{\infty} c_n x^n. \]

Use the equation \( f'' = f \) to get a relationship between the coefficients \( c_n \) and \( c_{n-2} \).

(ii) Use the relationships from (i) to express \( c_2 \) in terms of \( c_0 \) and \( c_2n+1 \) in terms of \( c_1 \).

(iii) Use (ii) to recognize that the power series for \( f \) must be obtained as a linear combination of the power series of \( e^x \) and \( e^{-x} \), yielding the general solution to \( y'' = y \).

(iv) Apply the same method to solve \( y'' = -y \).

2. Find the following areas. It may be helpful to sketch out what the regions look like.

(i) \( f(x) = \sin(x) \) and \( g(x) = x \) on the interval \([\pi/2, \pi]\).

(ii) The area enclosed by \( f(x) = x^2 \) and \( g(x) = 4x - x^2 \).

(iii) The region enclosed by \( f(x) = \frac{x}{1+x^2} \) and \( g(x) = \frac{x^2}{1+x^2} \).

(iv) The area enclosed by \( f(x) = 1/x, \ g(x) = x, \ h(x) = x/4 \) when \( x > 0 \).

3. Find the volume of the following solids of rotation:

(i) Spin the region \( y = x + 1, \ y = 0, \ x = 0, \) and \( x = 2 \) about the \( x \)-axis.

(ii) Rotate \( y = e^x, \ y = 0, \ x = -1, \) and \( x = 1 \) about the \( x \)-axis.

(iii) Turn \( y = x^2 \) and \( x = y^2 \) around the line \( y = 1 \).

(iv) Gyrate \( y = x^3, \ y = 0, \) and \( x = 1 \) about the line \( x = 2 \).