1. Let’s do a few more trigonometric substitutions, just to get a feel for what sort of integration problems they might be useful for:
   
   (i) Use the substitution \( y = 3 \sin(\phi) \) to compute \( \int \sqrt{9 - y^2} \, dy \).
   
   (ii) Use the substitution \( x = 2 \tan(\theta) \) to compute \( \int \frac{1}{x^2 + 4} \, dx \).
   
   (iii) Use the substitution \( z = 5 \sec(\psi) \) to compute \( \int \frac{1}{\sqrt{z^2 - 25}} \, dz \).

2. Here is some practice for integrals of rational functions:
   
   (i) \( \int \frac{x^4}{x - 1} \, dx \),
   
   (ii) \( \int \frac{y}{(y-3)(2y+1)} \, dy \),
   
   (iii) \( \int \frac{r^2 + t + 2}{r^2 - 1} \, dt \),
   
   (iv) \( \int \frac{1}{z^2 - 2z} \, dz \),
   
   (v) \( \int \frac{e^{2x}}{e^{2x} + 3e^x + 2} \, dx \),
   
   (vi) \( \int \frac{u + 2}{u^4 + 3u^3 + 3u^2 + u} \, du \),
   
   (vii) \( \int \frac{1}{1 + e^y} \, dy \),
   
   (viii) \( \int \frac{4v + 2}{v(v^2 + 1)^2} \, dv \).

   You may have needed to factor a quadratic polynomial, perhaps using the quadratic formula...

3. Let’s do something fun with polar coordinates!
   
   (i) Sketch the curve defined in polar coordinates by \( r = 1 - \cos(\theta) \).

   Feel free to ask your computer for help.
   
   (ii) Compute the integral

   \[ A = \frac{1}{2} \int_0^\pi r^2 \, d\theta \]

   where \( r \) is the function of \( \theta \) defined in (i).
   
   (iii) Informally explain in your own words why the quantity \( 2A \) is the area of the figure drawn in (i). It may be helpful to know the area of the sector of a circle is \( \frac{1}{2} r^2 \theta \). For somewhat a formal explanation, see here.

   This figure is called a cardioid. I think it’s rather pretty. Happy Valentines Day!