In recent joint work with Lars Andersson (AEI, Potsdam) we studied the stability, in the direction of cosmological expansion, of a family $n+1$ dimensional Lorentz cones over negative Einstein spaces of dimension $n > 3$. We used a variant of the constant mean curvature spatially harmonic (CMCSH) gauge condition introduced earlier. An important difference from the $3+1$ dimensional case treated previously is that now one may have a nontrivial moduli space of negative Einstein geometries. This makes it necessary to introduce a time-dependent reference metric which is used to define the spatially harmonic coordinate system that goes into the gauge.

By introducing scale factors defined in terms of the mean curvature and rewriting Einstein’s equations in terms of scale invariant geometric quantities we use suitable higher order energy estimates to prove that the scale invariant spatial metric decays asymptotically to an Einstein metric in the associated moduli space provided that the initial energy is sufficiently small. The argument assumes a certain stability property for the “background” Einstein metric of which no counterexamples are currently known.