Math 102 Fall 2008 Exam 2

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Instructions: This is a closed book, closed notes exam. Use of calculators is not permitted. You have fifty minutes. Do all 5 problems. Please do all your work on the paper provided. You must show your work to receive full credit on a problem. An answer with no supporting work or explanation will receive little to no credit.

Please print your name clearly here.

Print name: ______________________________

Upon finishing please sign the pledge below:
On my honor I have neither given nor received any aid on this exam.

Grader’s use only:

1. _______/10

2. _______/10

3. _______/10

4. _______/10

5. _______/10
1. [10 points]

Determine whether the following sequences converge or diverge. If they converge find the limit.

a) \( a_n = \sqrt[n]{\frac{n^2 + 1}{3n^2 + n + 7}} \)

b) \( a_n = \frac{\cos n}{\sqrt{n}} \)

c) \( a_n = \frac{n}{n + n} \)
2. [10 points]

Determine whether the following series converge or diverge. If they converge find the sum.

a) \[ \sum_{n=0}^{\infty} \frac{1+2^n}{3^n} \]

b) \[ \sum_{n=0}^{\infty} \sqrt[3]{3^n} \]
3. [10 points]

Determine if the following series converges or diverges

\[
\sum_{n=1}^{\infty} \frac{(-1)^n \sin n}{n^2}
\]

Make sure to give sufficient explanation to justify your answer.
4. [10 points]

How many terms do you need to add up in the series

\[ \sum_{n=1}^{\infty} \frac{1}{n^2 + \sqrt{n}} \]

so that the error is at most $10^{-3}$? Make sure you justify your answer clearly.
5. [10 points]
   (a) What is the power series of \( \frac{1}{1+x} \) and what is its \textbf{radius} of convergence?

   (b) What is the power series of \( \ln(1 + x) \)?

   (c) What is the power series of \( \ln(2 + x^2) \) and what is its \textbf{radius} of convergence?