Math 212 Spring 2008 Exam 1
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Instructions: This is a closed book, closed notes exam. Use of calculators is not permitted. You have one hour and fifteen minutes. Do all 7 problems. Please do all your work on the paper provided. You must show your work to receive full credit on a problem. An answer with no supporting work or explanation will receive little to no credit. Please print your name clearly here.

Print name: ______________________________________

Upon finishing please sign the pledge below:
On my honor I have neither given nor received any aid on this exam.

Grader’s use only:

1. _______/7

2. _______/15

3. _______/15

4. _______/18

5. _______/20

6. _______/15

7. _______/10
1. [7 points]
   Find the volume of the parallelepiped with sides $3i + 2j + k$, $i + k$ and $i + 2j + 3k$. 
2. [15 points]

Find the intersection of the line through the points (1, 0, 2) and (0, 1, −1) and the plane (through the origin) containing the two vectors $a = (1, 2, -1)$ and $b = (0, 3, 2)$. 
3. Consider the surface $S_1$ given by $\theta = \pi/2$ in cylindrical coordinates and the surface $S_2$ given by $\rho = 10$ in spherical coordinates.

(a) [10 points] Describe the intersection of $S_1$ and $S_2$ (a drawing would be nice).

(b) [5 points] Give a parametrization of the curve obtained in part (a).
4. An overheated penguin finds itself at location $(1, 0, 1)$. Suppose the temperature of the water is given by the function $T(x, y, z) = x^3 + e^{-yz}$.

(a) [8 points] In what direction should the penguin swim if it wants the temperature of the water to decrease fastest.

(b) [10 points] If the penguin swims in the direction $(1, 0, 4)$ (for a very short amount of time) will the temperature of the water increase or decrease. Explain!
5. [20 points]

Calculate, using the chain rule, the derivative of the composition \( f \circ g \) where 
\[
f(x, y) = (e^x + \sin(xy), y^2)
\]
and 
\[
g(u, v) = (\sin(u), u^2 + 2v)
\]
at the point \((u, v) = (0, 1)\) i.e. calculate \( \mathbf{D}(f \circ g)(0, 1) \).
6. [15 points]

A particle travels along the path \( c(t) = (\sin t, \cos t, t) \). When \( t = \pi \) it takes off in the tangential direction to the path and proceeds in a straight line. At what time \( t \) will the particle strike the plane \( x + 2z = 15 \).
7. [10 points]

Find the equation of the tangent plane to

\[ f(x, y) = x^2 + 2y^3 - 5 \]

at the point \((x_0, y_0, z_0) = (1, 1, -2)\).