1 Tensor Inoor:

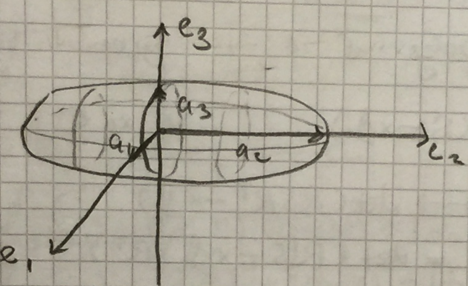
Let $S \in \mathrm{Mat}_{3\times3}(\mathbb{R})$: $S = S^T > 0$.

The equation $\langle x, Sx \rangle = 1$ defines an ellipsoid in $\mathbb{R}^3$ (all points $x \in \mathbb{R}^3$ which obey the eq-n).

(That is <u>the def.</u> of an ellipsoid)

<u>Cl.:</u> The eigenvectors of S point in the direction of the semi-major axes and the eigenvalues give the $(\text{lengths}^{-1})^2$.

<u>Pf.:</u>



Since $S = S^T > 0$, S may be orthogonally diagonalized w/ e-vals > 0.

$\Rightarrow S = O^T D O$ w/ $O \in O(3)$, $D = \mathrm{diag}(\lambda_1, \lambda_2, \lambda_3)$ w/ $\lambda_i > 0$.

Cl.: The coefficients of the characteristic poly. of S are invariant under $S \mapsto RSR^T$ $\forall\ R \in SO(3)$.

Pf.:

Hence we find

$$p(x) = \det(S - x\mathbb{1}) = \det(O^T D O - x\mathbb{1}) = \det(D - x\mathbb{1})$$

$$= (\lambda_1 - x)(\lambda_2 - x)(\lambda_3 - x)$$

Characterization of Euclidean Motions

Let $f: \mathbb{R}^n \circlearrowleft$ be a deformation.

Cl.: $(Df)(x) \in SO(n) \;\; \forall\, x \in \mathbb{R}^n \Rightarrow (Df)(x)$ does not dep. on x and $f(x) = (Df)(x) + f(0)$

Pf.: Assume $(Df)(x) \in SO(n) \;\; \forall\, x \in \mathbb{R}^n$.

Cl.: If $R \in O(n)$, then the columns of R form an ONB of $\mathbb{R}^n$.

Pf.: [redacted]

$\Rightarrow \{\partial_i f(x)\}_{i=1}^n$ is an ONB of $\mathbb{R}^n$.

Cl.: $(\partial_i \partial_j f)(x) = \underbrace{\langle (\partial_i \partial_j f)(x), \partial_k f(x) \rangle}_{=:\, \alpha_{ijk}(x)} (\partial_k f)(x)$

Pf.: [redacted]

Cl.: $\alpha_{ijk} = \alpha_{jik}$ **Pf.:** [redacted]

Cl.: $\alpha_{ijk} = -\alpha_{ikj}$

Pf.: [redacted]

Cl.: $\alpha_{ijk} = 0$

Pf.:

$\Rightarrow \partial_i \partial_j f = 0 \quad \forall \; i, j$

$\Rightarrow$ Df is a constant and $f(x) = f(0) + (Df)(x)$