Mechanics of Continua — HW #7

[Q1] The Buckling of a Pillar

(i) $M(x) = -P\, u(x)$

$$\Rightarrow u'' = -\frac{P}{EI} u \Rightarrow u = A \sin(k\cdot) \quad , k \equiv \frac{P}{EI}$$

B.C. $u(l) = 0 \Rightarrow A = 0$ (trivial) or

$$kl \in \pi\mathbb{Z} \Rightarrow \text{Smallest } P = \frac{EI}{(\pi/l)^2}$$

(ii) Cl.: Energy given by

$$F[u] = \frac{1}{2} EI \int_0^l u''^2 - \frac{1}{2} P \int_0^l u'^2$$

Pf.: $M = EI u''$

Note $\langle \sigma, \varepsilon \rangle = \sigma_{33} \varepsilon_{33} = \frac{M^2}{EI^2} x_1^2$

$$\Rightarrow \frac{1}{2} \int \langle \sigma, \varepsilon \rangle = \frac{M^2}{2EI}$$

$$\Rightarrow W = \frac{EI}{2} \int_0^l u''^2 \qquad \text{elastic energy}$$

Potential energy: $U = -Ph$

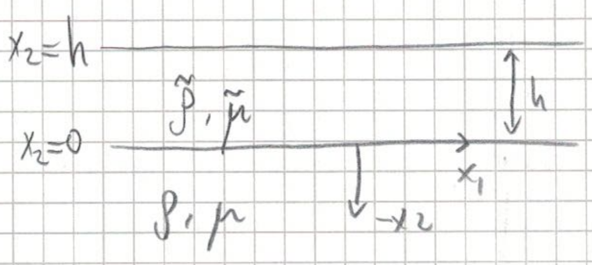
$$h = l - \int_0^l dx \frac{1}{\sqrt{1 + u'^2}} \approx \frac{1}{2} \int_0^l u'^2$$

$u(0) = u(l) = 0$

Euler-Lagrange eq-n for F: eq-n (1)

Love - Waves

Transversal surface waves.



ρ – density

μ – Elastic module

⋮

$$u_3 = \tilde{A}(x_2)\, e^{ik(x_1 - ct)} \qquad (\text{w/ or w/o } \sim)$$

Cl.: $\exists$ wave $\Longleftrightarrow$ $\tilde{c}_t < c_t$ and then c is given by

$$\operatorname{tg}\left(\sqrt{\left(\frac{c}{\tilde{c}_t}\right)^2 - 1}\, kh\right) = \frac{\mu}{\tilde{\mu}} \frac{\sqrt{1 - (c/c_t)^2}}{\sqrt{(c/\tilde{c}_t)^2 - 1}}$$

Pf.: $u(x) = u_3(x_1, x_2, t)\, e_3$

$$\lim_{x_2 \to -\infty} u = 0$$

Wave eq-n: $\partial_1^2 u_3 + \partial_2^2 u_3 = c_t^{-1} \partial_t^2 u_3$

I. No stress at $x_2 = h$.

II. Cont. of σ @ $x_2 = 0$.

III. Cont. of u_3 @ $x_2 = 0$.

Ansatz: $$u_3(x) = \begin{cases} A(x_2)\, e^{ik(x_1 - ct)} & x_2 < 0 \\ \tilde{A}(x_2)\, e^{ik(x_1 - ct)} & x_2 \in (0, h) \end{cases}$$