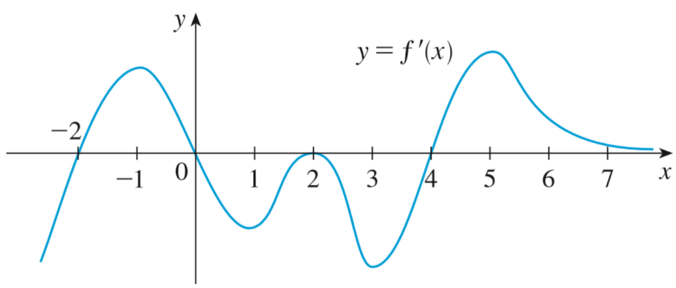
Homework 10

Calculus I - Math UN1101 (Section 02)

Due : April 22th

Part 1 Review

1. The figure shows the graph of the *derivative* of f' of a function f.



(a) On what intervals is f increasing or decreasing?

(b) For what values of x does f have a local maximum or minimum?

(c) Sketch the graph of f''.

(d) Sketch a possible graph of f.

2. Find the local and absolute extreme values of the function on the given interval.

(a)

$$f(x) = x\sqrt{1 - x}, \quad [-1, 1]$$

(b)

$$f(x) = \frac{3x - 4}{x^2 + 1}, \quad [-2, 2]$$

(c)

$$f(x) = \sqrt{x^2 + x + 1}, \quad [-2, 1]$$

Part 2 Ongoing Lecture Material

1. Determine whether the statement is true or false. If it is true, explain why. If it is false, explain why and give an example that disproves the statement.

(a)

If f and g are continuous on $[a, b]$, then

$$\int_a^b [f(x) + g(x)]\, dx = \int_a^b f(x)\, dx + \int_a^b g(x)\, dx$$

(b)

If f and g are continuous on $[a, b]$, then

$$\int_a^b [f(x)g(x)]\, dx = \left(\int_a^b f(x)\, dx\right)\left(\int_a^b g(x)\, dx\right)$$

(c)

If f is continuous on $[a, b]$, then

$$\int_a^b 5f(x)\, dx = 5\int_a^b f(x)\, dx$$

(d)

If f is continuous on $[a, b]$, then

$$\int_a^b xf(x)\, dx = x\int_a^b f(x)\, dx$$

(e)

If f is continuous on $[a, b]$ and $f(x) \geqslant 0$, then

$$\int_a^b \sqrt{f(x)}\, dx = \sqrt{\int_a^b f(x)\, dx}$$

(f)

If f' is continuous on $[1, 3]$, then $\int_1^3 f'(v)\, dv = f(3) - f(1)$.

2. Evaluate the following integrals using Theorem 9.35. (Change of variables)

$$\int_1^2 \frac{e^{1/x}}{x^2}\,dx$$

$\int_0^1 xe^{-x^2}\,dx$

$$\int_0^4 \frac{x\,dx}{\sqrt{1+2x}}$$

$$\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}}$$

$$\int_0^{1/2} \frac{\sin^{-1} x}{\sqrt{1-x^2}}\,dx$$

$$\int_0^1 \frac{e^z+1}{e^z+z}\,dz$$

3. Evaluate the following integrals using Theorem 9.41. (Integration by parts)

$$\int_0^{1/2} x\cos \pi x\,dx$$

$\int_0^1 (x^2+1)e^{-x}\,dx$

$\int_0^1 t\cosh t\,dt$

$$\int_4^9 \frac{\ln y}{\sqrt{y}}\,dy$$

$\int_1^3 r^3 \ln r\,dr$

$\int_0^{2\pi} t^2 \sin 2t \, dt$

4. Evaluate the integral by interpreting it in terms of areas.

$\int_0^1 \left(x + \sqrt{1 - x^2}\right) dx$