Calculus I: Homework 11

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Problem 1: Solve the following integrals. When the integration bounds are omitted ($\int$), they are meant to be taken from a until b $\int_a^b$, for some two real numbers a and b.

$$\int \frac{12x}{\sqrt{2x^2+5}}\,dx$$

$$\int 2x\sin(3x)\,dx$$

$$\int_2^3 \frac{3x^2+2x+1}{x}\,dx$$

$$\int \tan x\,dx$$

$$\int \frac{\arctan\sqrt{x}}{\sqrt{x}}\,dx$$

$$\int_{-2}^{1} |x|\,dx.$$

$$\int x^2\log(x)\,dx$$

$$\int \frac{3\cos\left(\log(x)\right)}{x}\,dx$$

Problem 2: This is more a review problem for the entire semester. Please explain briefly your sketch. A sketch alone will not receive full credits.

Sketch the graph of the function f that has the following properties: Please label all asymptotes

- $f(x)$ is continuous on its entire domain, which is all x except $x = 2$.
- $\lim_{x \to -\infty} f(x) = -\infty$ and $\lim_{x \to \infty} f(x) = 3$.
- $\lim_{x \to 2} f(x) = \infty$.
- $f'(x)$ is continuous at all x except $x = -1$, $x = 2$, and $x = 5$.
- $f'(x) > 0$ for $x < -1$ and for $0 < x < 2$ and for $4 < x < 5$ and for $x > 5$.
- $f'(x) < 0$ for $-1 < x < 0$ and for $2 < x < 4$.
- $\lim_{x \to -1^-} f'(x) = 3$ and $\lim_{x \to -1^+} f'(x) = -3$.
- $\lim_{x \to 5} f'(x) = \infty$.
- $f''(x) > 0$ for $-4 < x < -1$ and for $-1 < x < 2$ and for $2 < x < 5$.
- $f''(x) < 0$ for $x < -4$ and for $x > 5$.
- $f(-4) = -1$, $f(-1) = 4$, $f(0) = 2$, $f(4) = -2$, and $f(5) = 0$.

Problem 3: Let's find other ways of computing area

1- Is the following integral equality true or false?

$$\int_{-1}^{x} \sqrt{1-y^2}\,\mathrm{d}y = \frac{\pi}{4} + \frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\arcsin(x) \qquad (x \in (-1,1))$$

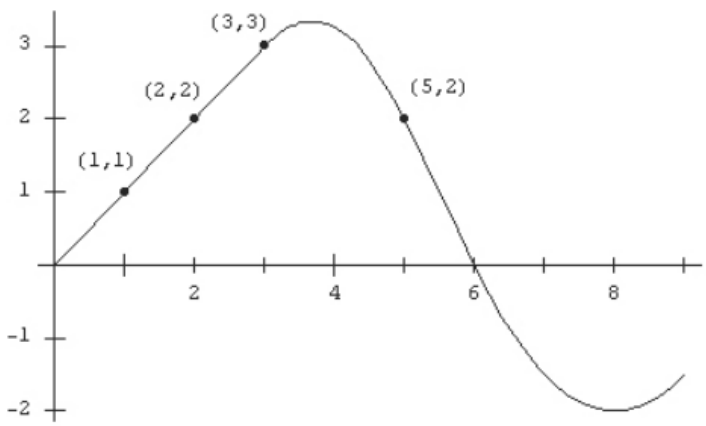
2- If true, **use the equality above** to compute the area of a circle of radius 1.

Problem 4: This problem is a good application of calculus in physics.
Consider the following graph.

This is the graph of the velocity of a particle that we denote as v(t) in meters per second.
Let s(t) (graph not shown) be the function of the **position** of the particle.

We have s(0) = 1 m

Please answer all questions with a clear explanation.
BACKGROUND INFO: The velocity function is the derivative of the position and the acceleration function is the derivative of the velocity function (all functions of time)



a) What is the particle's velocity at t= 5?
b) Is the acceleration of the particle at time t=5 positive or negative?
c) What is the particle's position at t=3?
d) At what time during the first 9 seconds does the position s have its largest value
e) Approximately when is the acceleration zero?
f) When is the particle moving **toward** the origin? **Away** from the origin?
g) On which side (positive or negative) of the origin does the particle lie at t= 9?
h) The integral of v(t) from 0 to 6 is 11.5
   The integral of v(t) from 6 to 9 is -4.5
   Find the total distance traveled by the particle the first 9 seconds.

Problem 5: State whether the following statements are True or False. If you think a statement is false, give a counter example. If it is true, you can just state it is. If you write a formal proof, you will get extra credit!

a) If f is continuous on $[a, b]$, then

$$\sqrt{\int_a^b f(x)\,dx} = \int_a^b \sqrt{f(x)}\,dx$$

b) If f and g are continuous on $[a, b]$, then

$$\int_a^b f(x) \pm g(x)\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

c) [3 points] $\int_{-1}^{3} 2x - x^2\,dx$ represents the area between the curve

$$y = 2x - x^2$$

the x-axis, $x = -1$ and $x = 3$.

d) If f is continuous on $[a, b]$, then

$$\frac{d}{dx}\left(\int_a^b f(x)\,dx\right) = f(x).$$