Functions

Exercise 1
Give an example of a function $f : \mathbb{R} \to \mathbb{R}$ which is strictly monotone increasing but not injective.

Exercise 2
Let $f : A \to B$ is a function with $|A| = 3$. Show that the following holds:
1. If $f$ is injective, then $|B| \geq 3$.
2. If $f$ is surjective, then $|B| \leq 3$.
3. If $f$ is bijective, then $|B| = 3$.

Exercise 3
Find $f \circ g$, and $g \circ f$ and their appropriate domains and co-domains for the following formulae:
1. $f(x) = \sqrt{x + 1}$, $g(x) = \sin x$
2. $f(x) = e^x$, $g(x) = -x^2 + 4x$
Express each function in the form $f \circ g$.
4. $\sin^2(\frac{1}{x})$
5. $\sqrt{2|x|}$
6. $5\sin(x) + 6$

Exercise 4

Sketch the graph and describe the image of each function $\mathbb{R} \to \mathbb{R}$:

1. $f(x) = -(x - 3)^2 + 4$
2. $f(x) = 4\sin(x - \frac{\pi}{2})$
3. $f(x) = e^{-x} + 5$

Draw the set of all points $(x, y) \in \mathbb{R}^2$ satisfying the following equations:

4. $|x| + |y| = 1$
5. $xy = 0$
6. $x^2 = y^2$

Limits

Exercise 5

Determine whether the sequence converges or diverges. If it converges, find the limit.

1. $\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \cdots \}$
2. $\{4, -1, \frac{1}{4}, -\frac{1}{16}, \frac{1}{64}, \cdots \}$
3. $\{\sin(0), \sin(\frac{\pi}{2}), \sin(\pi), \sin(\frac{3\pi}{2}), \sin(2\pi), \cdots \}$
4. $\{\frac{1}{2}, \frac{4}{3}, \frac{9}{4}, \frac{16}{5}, \frac{25}{6}, \cdots \}$

Exercise 6

Find the limit of the following functions. If the limit does not exist, explain why. If the limit diverges, then determine whether it is $\infty$ or $-\infty$:
1. \( \lim_{x \to 0} \frac{x^2 - 1}{x - 1} \)

2. \( \lim_{x \to \infty} \frac{x^2 - 1}{x - 1} \)

3. \( \lim_{x \to 0} 2^x \)

4. \( \lim_{x \to \infty} \frac{\sin(5x)}{x} \)

5. \( \lim_{x \to 0} \cos(3x) \cdot \sin\left(\frac{1}{x}\right) \)