Calculus 1–Section 2–Spring 2019–HW2

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Functions

Exercise 1

Give an example of a function $f : \mathbb{R} \to \mathbb{R}$ which is strictly monotone increasing but not injective.

Exercise 2

Let $f : A \to B$ is a function with $|A| = 3$. Show that the following holds:

1. If f is injective, then $|B| \geq 3$.
2. If f is surjective, then $|B| \leq 3$.
3. If f is bijective, then $|B| = 3$.

Exercise 3

Find $f \circ g$, and $g \circ f$ and their appropriate domains and co-domains for the following formulae:

1. $f(x) = \sqrt{x+1}$, $g(x) = \sin x$
2. $f(x) = e^x$, $g(x) = -x^2 + 4x$

Express each function in the form $f \circ g$.

4. $\sin^2(\frac{1}{x})$
5. $\sqrt{2|x|}$

6. $5\sin(x) + 6$

Exercise 4

Sketch the graph and describe the image of each function $\mathbb{R} \to \mathbb{R}$:

1. $f(x) = -(x-3)^2 + 4$
2. $f(x) = 4\sin(x - \frac{\pi}{2})$
3. $f(x) = e^{-x} + 5$

Draw the set of all points $(x, y) \in \mathbb{R}^2$ satisfying the following equations:

4. $|x| + |y| = 1$
5. $xy = 0$
6. $x^2 = y^2$

Limits

Exercise 5

Determine whether the sequence converges or diverges. If it converges, find the limit.

1. $\{\frac{1}{2},\ \frac{1}{4},\ \frac{1}{6},\ \frac{1}{8},\ \cdots\}$
2. $\{4,\ -1,\ \frac{1}{4},\ -\frac{1}{16},\ \frac{1}{64},\ \cdots\}$
3. $\{\sin(0),\ \sin(\frac{\pi}{2}),\ \sin(\pi),\ \sin(\frac{3\pi}{2}),\ \sin(2\pi),\ \cdots\}$
4. $\{\frac{1}{2},\ \frac{4}{3},\ \frac{9}{4},\ \frac{16}{5},\ \frac{25}{6},\ \cdots\}$

Exercise 6

Find the limit of the following functions. If the limit does not exist, explain why. If the limit diverges, then determine whether it is ∞ or $-\infty$:

1. $\lim_{x \to 0} \frac{x^2-1}{x-1}$

2. $\lim_{x \to \infty} \frac{x^2-1}{x-1}$

3. $\lim_{x \to 0} 2^{2^x}$

4. $\lim_{x \to \infty} \frac{sin(5x)}{x}$

5. $\lim_{x \to 0} \cos(3x) \cdot \sin(\frac{1}{x})$