Calculus 1–Section 2–Spring 2019–HW3

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February 11th, 2019

Exercise 1

Sketch the graph of a function f(x) with the following properties. There are many possible answers.

- First function

$$\lim_{x\to+\infty} f(x) = 3;$$

$$\lim_{x\to2^-} f(x) = -\infty;$$

$$\lim_{x\to2^+} f(x) = +\infty;$$

$$\lim_{x\to-2^-} f(x) = 3;$$

$$\lim_{x\to-2^+} f(x) = 1;$$

$$f(-4) = 2.$$

- Second function

$$\lim_{x\to+\infty} f(x) = 0;$$

$$\lim_{x\to-\infty} f(x) = -1;$$

$$\lim_{x\to5^-} f(x) = 4;$$

$$\lim_{x\to5^+} f(x) = 4;$$

$$\lim_{x\to-1^-} f(x) = +\infty;$$

$$\lim_{x\to-1^+} f(x) = -\infty;$$

$$f(0) > 0.$$

Exercise 2

Find all the points where f(x) is continuous:

- $f(x) = \log\left(\tan^3(2x)\right);$
- $f(x) = \begin{cases} 2 & x = 1, x = -1; \\ \dfrac{x^2-1}{|x|-1} & x \neq 1, -1. \end{cases}$

Exercise 3

Find each of the following limits, with justification. If the limit does not exist, explain why. If the function approaches infinity at the point from left or right then explain whether it is +∞ or −∞. Note: $\lim_{x\to+\infty} f(x) = \lim_{t\to 0^+} f\left(\frac{1}{t}\right).$

- $\lim_{x\to 0} \frac{\tan(\frac{1}{2}x^2)}{x};$
- $\lim_{x\to+\infty} \frac{(\sqrt{x}+x)(x-2)}{1+x\sqrt{x}};$
- $\lim_{x\to+\infty} \cos\left(e^{-x^2}2^{\sin x}\right);$
- $\lim_{x\to 1^-} \left(\frac{x^3+2}{(x-1)(x-2)} - x\right);$
- $\lim_{x\to 5^+} \left(\frac{x-5}{x} \cdot \frac{e^x}{x^2-6x+5}\right);$
- $\lim_{x\to+\infty} \left(\sqrt{x^2+8x} - x\right);$
- $\lim_{x\to 0} \log_2\left(\sin^2\frac{\pi}{x} + 1\right);$
- $\lim_{x\to 0} \sqrt{x}\cos\left(\sin\frac{1}{x}\right);$
- $\lim_{x\to -1} \sin\left(\frac{\pi(x-1)}{x^2+1}\right);$
- $\lim_{x\to\pi} \frac{\cos x + \sin x + 1}{x - 3\pi};$

Exercise 4

Find a formula for f′ (x) using the limit definition of the derivative. Show the steps of your computation.

- $f(x) = x^2$;
- $f(x) = \dfrac{x}{x^2 - 4}$;
- $f(x) = x + \sqrt{x}$;

Exercise 5

a) Find an example of functions f(x) and g(x) such that $\lim_{x \to 0} f(x)$ and $\lim_{x \to 0} g(x)$ does not exist but $\lim_{x \to 0} f(x)g(x)$ exists.

b) Find an example of functions f(x) and g(x) such that $\lim_{x \to 0} f(x)$ and $\lim_{x \to 0} g(x)$ does not exist but $\lim_{x \to 0} (f(x) + g(x))$ exists.