Calculus 1 Section 2 HW4 Part1—REVIEW OF PRECALC

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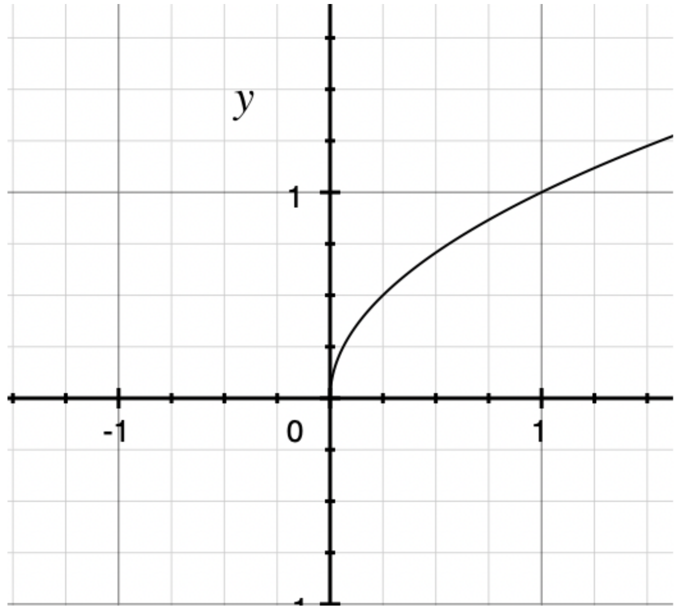
1. The equation, $4x^2 - 6x + 10 = 0$, has

- (a) four real solutions
- (b) two real solutions
- (c) three real solutions
- (d) no real solutions
- (e) one real solution

2. Simplify $[a^4 - 4a^2 + 4]^{1/2}$

3. Simplify $$\frac{\dfrac{x+1}{(x^2 - 2x - 3)}}{\dfrac{x - 3}{x + 1}}$$

The graph



4. would best approximate the graph of which of the following equations for on the interval [-1.5,1.5]?

- (a) $y = x^3$
- (b) $y^2 + x^2 = 1$
- (c) $y = x^2$
- (d) $y = x$
- (e) $y = x^{1/2}$

5. Find the equation of the line perpendicular to $y = x/3 + 1$ through the point (3,2).

- (a) $y = x/3 + 7$
- (b) $y = x/3 + 1$
- (c) $y = 3x - 7$
- (d) $y = -x/3 + 3$
- (e) $y = -3x + 11$

6. The line containing the points (1,1) and (1,-1) is perpendicular to the line

- (a) $y = 2x + 1$
- (b) $y = 5$
- (c) $y - 1 = x - 1$
- (d) $y = -x - 1$
- (e) $x = 5$

7. If $2(2x-3) + 5(x + 1) = 6x-7$, what is x?

8. If $x = 8$, find the largest value of y which satisfies $y^2x + yx^2 + 128 = 0$.

- (a) There is no largest value.
- (b) 4
- (c) 8
- (d) -8
- (e) -4

9. Find the largest solution of $x^3 + 4x^2 + 3x = 0$.

10. If $\log(x) = 4$ then $\log(x^2)$ is

11. $\log_2(3xy^2)$ is equal to

12. $\log(x^2 - 2x + 1) > \log(25)$ reduces to

13. If $h(x) = x^2$, $g(x) = x+1$, and $f(x) = x + 3$, then what is $h(g(f(0)))$?

14. If $h(x) = x^3$, $g(x) = x^2+1$ and $f(x) = x + 1$, then $h(g(f(0))) + f(g(h(0)))$ is:

15. If $f(x)=3x^2 + 3$, what is $f(f(a))$?

16. The expression $-18 < -6x < 12$ is equivalent to which of the following?

- (a) $3 > x > -2$
- (b) $-2 > x > -3$
- (c) $3 > x > 2$
- (d) $2 > x > -3$
- (e) All of the other answers are incorrect.

17. The inequality $x^3 < 2x^2 - x$ reduces to

18. The expression $(x + 1)(x + 2) > 0$ is equivalent to which of the following?

- (a) $x > -1$
- (b) $x < -2$
- (c) $x < -2$ or $x > -1$
- (d) All of the other answers are incorrect.
- (e) $-1 > x > -2$

19. If $3x + 4y = 7$ and $5x - 4y = 1$, find x and y.

20. If $x - y = 4$ and $4x - y = 1$, then $3xy$ is

21. If $y + 3x + 2 = 0$ and $y = x^2$, then there is a solution with x given by

22. The radian measure of an angle of 60 degrees is

23. If $\sin(a + b) = 1$ and $\tan(a) = 0$, then $\tan(b)$ is

Ziad Saade

Calculus 1 Section 2 HW4 Part1—ACTUAL CALC1 HW

Homework 4: Calculus 1

Continuity and its properties, the intermediate value theorem, introduction to derivatives

Problem 1:

Part a:

Use the Intermediate Value Theorem to show that the given equation has at least one solution in the indicated interval. You are NOT asked to find the solution only show that at least one must exist in the indicated interval.

$a)\ 25 - 8x^2 - x^3 = 0 \quad for\ x\ in\ [-2,4]$

$b)\ w^2 - 4\log(5w+2) = 0 \ \ for\ w\ in\ [0,4]$

$c)\ 4t + 10e^t - e^{2t} = 0 \quad for\ t\ in\ [1,3]$

Part b:

True or False? If True, explain thoroughly. If False, give a counterexample by drawing an accurate graph of what the function would look like or by explaining.

f is a function continuous on [-1,1] such that f(-1) = 0, f(0) = 1 and f(1) = 0

a) There exists at least one real value a in [-1,0] such that f(a) = 0.5
b) There exists a unique real value b in [0,1] such that f(b) = 0.5
c) The equation f(x) = 0.5 has at least two solutions in [-1,1]
d) The equation f(x) = 0.5 has exactly two solutions in [-1,1]

Problem 2:

Let a be a real value, and f is a function defined on an open interval containing 0. The limit of f as x approaches 0 is 0. Which of these propositions are true? Why?

In this question, we use the shortcuts: $\forall$ means "for all", $\exists$ means "there exists"
(Note this problem is mainly to test your ability to understand the notations)

1. $\forall \varepsilon > 0,\ \exists \eta > 0,\ \forall x \le \eta,\ |f(x)| \le \varepsilon$

2. $\forall \varepsilon \in (0, 1),\ \exists \eta \in (0, 1),\ 0 < |x| \le \eta \Rightarrow |f(x)| \le \varepsilon$

3. $\forall \varepsilon > 0$, $\exists \eta > 0$, $\forall x \in [-\eta, \eta]$, $|f(x)| \leq \varepsilon$

4. $\forall \varepsilon > 0$, $\exists \eta > 0$, $\forall x \in [-\eta, 0) \cup (0, \eta]$, $|f(x)| < \varepsilon$
5. $\forall n \in N*$ (interval not containing 0), $\exists \eta > 0$, $\forall x \in [-\eta, 0] \cup (0, \eta)$, $|f(x)| < (1/n)$

6. $\forall n \in N$, $\exists m \in N$, $|x| \leq (1/m) \Rightarrow |f(x)| \leq (1/n)$

7. $\forall n \in N$, $\exists m \in N$, $0 < |x| \leq (1/m) \Rightarrow |f(x)| \leq (1/n^2)$

Problem 3:
This is a problem where your reasoning is tested. The problem has one question but you have to take initiatives and try to find the steps leading to the solutions.

The definition of a derivative of a function f at a point x is a limit

$$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

Use this definition to show that the derivative of the product fg is such that

$$(fg)' = f'g + fg'$$

You just derived one of the most important theorems related to derivatives.

HINT: Firs step is to express (fg)' using the definition given.

Problem 4:

a) Determine the values for which f is continuous:

$$f(0) = 0 \text{ and } f(x) = x + \frac{\sqrt{x^2}}{x}$$

b) Let f be a function defined on [0,1]

$$f(x) = \begin{cases} 0 & if\ x = 0 \\ x + \dfrac{x\log(x)}{1-x} & if\ 0 < x < 1 \\ 0 & if\ x = 1 \end{cases}$$

- Show that f is continuous on [0,1]

c) $f(x) = \begin{cases} \frac{\sin(x)}{x} & if\ x < 0 \\ 1 & if\ x = 0 \\ x^2 + 1 & if\ x > 0 \end{cases}$

f is a function defined on R. Is it continuous on R?

d) $f(x) = \begin{cases} \frac{1}{1+x} & if\ 0 \leq x < 0.5 \\ 2x + mx^2 & if\ 0.5 \leq x < 1 \end{cases}$

Find the values of m (in R), if they exist, so that the function f is continuous.