Calculus 1 – Spring 2019 Section 2–HW6

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1 Review

Exercise 1. Find an equation for the line that passes through the point $(2,2)$ and is parallel to the vertical axis.

Exercise 2. Draw the shape formed by all points $(x,y)\in\mathbb{R}^2$ obeying the equation

$$x^2+\frac{y^2}{4} = 1$$

as accurately as you can.

Exercise 3. Draw the region of $\mathbb{R}^2$ of all points $(x,y)\in\mathbb{R}^2$ obeying the constraint $|x|<1$ and $y>3$.

Exercise 4. Express $\tan(2x)$ in terms of some arithmetic operations in $\tan(x)$.

Exercise 5. Express $\cos(x+y)$ in terms of $\cos(x)$, $\sin(x)$, $\cos(y)$ and $\sin(y)$.

Exercise 6. If the sketch of the graph of $f(x)=x^2$ restricted to positive x is rotated by $\frac{\pi}{2}$-radians clock-wise, we get the sketch of the graph of a function given by which formula?

2 Exercises pertaining to new material

Exercise 7. Calculate $\lim_{\varepsilon\to 0}(1+2\sin(\varepsilon))^{\cot(\varepsilon)}$ using the hospital rule (eventually).

Exercise 8. Calculate f' if $f(x)=\sin(x)^2+\cos(x)^2$ for all $x\in\mathbb{R}$.

Exercise 9. Determine whether the following functions is differentiable (and where) and whether its derivative is continuous (and if so where):

$$\begin{aligned} f:\mathbb{R} &\to \mathbb{R} \\ x &\mapsto \begin{cases} x^2\sin\left(\frac{1}{x}\right) & x\neq 0 \\ 0 & x=0 \end{cases} \end{aligned}$$

Exercise 10. Let $f(x):=(x-\sqrt{x})^2$. Find f' and f''.

Exercise 11. Find two (different) functions $f:\mathbb{R}\to\mathbb{R}$ which obey $f''=\alpha^2 f$ and $f(0)=1$ for some $\alpha\in\mathbb{R}$. Recall f'' is the *second* derivative of f, i.e., it is the derivative of f'. Also recall that $\exp'=\exp$ and use the chain rule.

Exercise 12. Find a function f such that $f'=-f^2$ and such that $f(1)=1$.

Exercise 13. Let $f(x):=\left(\log\left(\frac{1}{\sqrt{\tanh(x)}}\right)\right)^3$ for all x for which this makes sense (see the definition of tanh in the lecture notes). Calculate f'.

Exercise 14. Find all points at which the *derivative* of the following functions is zero. Such points are called *critical points*. Say (if you can) whether these points which you find are (global or local) maxima or minima.

1. $f:\mathbb{R}\to\mathbb{R}$ given by $x\mapsto x^2$.
2. $f:\mathbb{R}\to\mathbb{R}$ given by $x\mapsto\sinh(x)^2$.

3. $f : \mathbb{R} \to \mathbb{R}$ given by $x \mapsto \tanh'(x)$ (i.e. the *derivative* of tanh).

Exercise 15. Do the hypothesis of the mean value theorem (see lecture notes) hold for $f(x) = 3x^2 - \frac{2}{x}$ on the interval $[-1, 1]$?

Exercise 16. Johannes is driving on I95 which has a speed limit of $55mph$ southwards from New York City. At 10am he was 110 miles away from NYC and at 1pm he was 290 miles away from NYC. Provide evidence to the court that Johannes was speeding, by assuming that the function f : time $\to$ miles away from NYC is a continuous function and differentiable, and its derivative is precisely the *instantaneous speed* of Johannes.