Calculus 1–Section 2–Spring 2019–HW3 Solutions

Mat Hillman

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Review

Exercise 1

Part a: False; f has a limit at x_0 if and only if

(i) the left and right limits of f at x_0 exist

(ii) the left and right limits of f at x_0 agree

that is,

$$\lim_{x \to x_0^-} f(x) = \lim_{x \to x_0^+} f(x)$$

So to give a counter example, we simply need to break

Consider this condition

$$f(x) = \begin{cases} -2 & \text{if } x \leq 0 \\ 2 & \text{if } x > 0 \end{cases}$$

Then

$$\lim_{x \to 0^+} f(x) = 2 \neq -2 = \lim_{x \to 0^-} f(x)$$

Part b: True; Notice that for all x, we have that

$$-1 \leq \cos(x) \leq 1$$

$$\Rightarrow \frac{-1}{2} \leq \frac{-\cos(x)}{2} \leq \frac{1}{2}$$

$$\Rightarrow \frac{1}{2} \leq 1 - \frac{\cos(x)}{2} \leq \frac{3}{2}$$

$$\Rightarrow \frac{1}{\sqrt{2}} \leq \sqrt{1 - \frac{\cos(x)}{2}} \leq \frac{\sqrt{3}}{\sqrt{2}}$$

contin. function

Part c: True; Consider $f(x) = \cos\left(\frac{\pi}{2}x\right) - \log_2\left(x - \frac{1}{2}\right)$

Recall that $\log_2\left(x-\frac{1}{2}\right) < 0$ for $x < \frac{3}{2}$.

So

$$f(1) = \cos\left(\frac{\pi}{2}\right) - \log_2\left(\frac{1}{2}\right) = 0 - \log_2\left(\frac{1}{2}\right) > 0$$

Also

$$f\left(\frac{3}{2}\right) = \cos\left(\frac{3\pi}{4}\right) - \log_2(1) = \cos\left(\frac{3\pi}{4}\right) < 0$$

So by the Intermediate Value Theorem, there exists x_0 st $1 < x_0 < \frac{3}{2}$ w/ $f(x_0) = 0$

$\Rightarrow \cos\left(\frac{x_0\pi}{2}\right) = \log_2\left(x_0 - \frac{1}{2}\right)$ for some x_0

$\Rightarrow$ the eqn has a root

Part d: False; Notice that

$$f(0) = c \cdot 2^{0^2+1} = c \cdot 2 = -2$$

for $c = -1$. Since $0 \le 1 - \cos(x) \le 2$, we see that

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{1-\cos(x)}{x^2} \ge 0$$

$\Rightarrow \lim_{x\to 0^-} f(x) \ne -2$

$\Rightarrow f$ is not continuous at 0.

Part e: False; We compute the limit

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{\sqrt{n^2-3}}{n+1}$$

$$= \lim_{n\to\infty} \frac{\frac{1}{n}\sqrt{n^2-3}}{\frac{1}{n}(n+1)}$$

$$= \lim_{n\to\infty} \frac{\sqrt{1-\frac{3}{n^2}}}{1+\frac{1}{n}}$$

$$= \frac{\lim_{n\to\infty}\sqrt{1-3/n^2}}{\lim_{n\to\infty} 1+1/n}$$

$$= \frac{\sqrt{\lim_{n\to\infty} 1-3/n^2}}{\lim_{n\to\infty} 1+1/n}$$

$$= \frac{\sqrt{1}}{1}$$

$$= 1$$

Part f: False; Consider $f(x) = |x|$.

To see that f is cts at $x_0 = 0$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} |x| = 0 = f(0) = f(\lim_{x\to 0^+} x)$$

To see that f is not diff'le at $x_0 = 0$

$$\lim_{h\to 0^-} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0^-} \frac{|h|}{h} = \lim_{h\to 0^-} \frac{-h}{h} = -1$$

$$\neq$$

$$\lim_{h\to 0^+} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0^+} \frac{|h|}{h} = \lim_{h\to 0^+} \frac{h}{h} = 1$$

<u>Exercise 2</u>

Vertical Asymptotes can only occur where the denominator is undefined. In our case this is at $x = \pm 1$.

$$\lim_{x \to \pm 1} f(x) = \lim_{x \to \pm 1} \frac{x^2 - x + 3}{1 - x^2}$$

Notice that

$$\lim_{x \to 1} x^2 - x + 3 = 3$$

$$\lim_{x \to -1} x^2 - x + 3 = 5$$

and

$\lim_{x \to 1^-} 1 - x^2$ approaches 0 from the right

$\lim_{x \to 1^+} 1 - x^2$ approaches 0 from the left

$\lim_{x \to -1^-} 1 - x^2$ approaches 0 from the left

$\lim_{x \to -1^+} 1 - x^2$ approaches 0 from the right

$\Rightarrow f$ has vertical asymptotes at $x = \pm 1$ w/

$$\lim_{x \to 1^+} f(x) = -\infty = \lim_{x \to -1^-} f(x)$$

$$\lim_{x \to -1^+} f(x) = \infty = \lim_{x \to 1^-} f(x)$$

There exist horizontal asymptotes if

$$\lim_{x \to \pm\infty} f(x)$$

exists, ie, is finite. So we compute these limits

$$\lim_{x\to\pm\infty} f(x) = \lim_{x\to\pm\infty} \frac{\frac{1}{x^2}(x^2 - x + 3)}{\frac{1}{x^2}(1-x^2)}$$

$$= \lim_{x\to\pm\infty} \frac{1 - 1/x + 3/x^2}{\frac{1}{x^2} - 1}$$

$$= \frac{\lim_{x\to\pm\infty} 1 - 1/x + 3/x^2}{\lim_{x\to\pm\infty} 1/x^2 - 1}$$

$$= \frac{1}{-1}$$

$$= -1$$

$\Rightarrow$ f has a horizontal asymptote along $y=-1$ in both the positive direction and negative direction.

Exercise 3

(1) For $f(x) = \dfrac{e^{x^2}}{\sin 3x}$,

$$f'(x) = \frac{(e^{x^2})' \cdot \sin 3x - e^{x^2} \cdot (\sin 3x)'}{\sin^2 3x} \quad \text{[quotient rule]}$$

$$= \frac{e^{x^2} \cdot 2x \cdot \sin 3x - e^{x^2} \cdot \cos 3x \cdot 3}{\sin^2 3x} \quad \text{[chain rule]}$$

$$= \frac{e^{x^2}(2x \sin 3x - 3\cos 3x)}{\sin^2 3x}.$$

Remark. Using the reciprocal trigonometric functions, you may rewrite this as the *seemingly* (but not really) simpler expression

$$f'(x) = e^{x^2} \csc 3x \cdot (2x - 3\cot 3x).$$

(2) For $f(x) = \tan x \cdot \log_2^2 x$ (note that $\log_2^2 x$ means $(\log_2 x)^2$),

$$f'(x) = (\tan x)' \cdot \log_2^2 x + \tan x \cdot (\log_2^2 x)' \quad \text{[product rule]}$$

$$= \sec^2 x \cdot \log_2^2 x + \tan x \cdot 2\log_2 x \cdot (\log_2 x)' \quad \text{[chain rule]}$$

$$= \sec^2 x \cdot \log_2^2 x + \tan x \cdot 2\log_2 x \cdot \left(\frac{\ln x}{\ln 2}\right)' \quad \text{[change of base formula]}$$

$$= \sec^2 x \cdot \log_2^2 x + \tan x \cdot 2\log_2 x \cdot \frac{1}{\ln 2} \cdot \frac{1}{x}$$

$$= \sec^2 x \cdot \log_2^2 x + \frac{2\tan x \cdot \log_2 x}{\ln 2 \cdot x}.$$

(3) For $f(x) = (\ln(x^2))^{x^5+3x}$, logarithmic differentiation is most convenient. Setting $y = (\ln(x^2))^{x^5+3x} = (2\ln x)^{x^5+3x}$, we have

$$\ln y = \ln\left((2\ln x)^{x^5+3x}\right) = (x^5+3x)\ln(2\ln x).$$

Differentiating implicitly with respect to x gives

$$\begin{aligned}
\frac{1}{y}\cdot\frac{dy}{dx} &= \left[(x^5+3x)\ln(2\ln x)\right]' \\
&= (x^5+3x)'\ln(2\ln x) + (x^5+3x)\cdot[\ln(2\ln x)]' \quad \text{[product rule]} \\
&= (5x^4+3)\ln(2\ln x) + (x^5+3x)\cdot\frac{1}{2\ln x}\cdot[2\ln x]' \quad \text{[chain rule]} \\
&= (5x^4+3)\ln(2\ln x) + (x^5+3x)\cdot\frac{1}{2\ln x}\cdot\frac{2}{x} \\
&= (5x^4+3)\ln(2\ln x) + \frac{(x^4+3)}{\ln x}.
\end{aligned}$$

Therefore, $\frac{dy}{dx} = y\left[(5x^4+3)\ln(2\ln x) + \frac{(x^4+3)}{\ln x}\right]$, i.e.,

$$\begin{aligned}
f'(x) &= (2\ln x)^{x^5+3x}\left[(5x^4+3)\ln(2\ln x) + \frac{(x^4+3)}{\ln x}\right] \\
&\text{or } (\ln(x^2))^{x^5+3x}\left[(5x^4+3)\ln(\ln(x^2)) + \frac{(x^4+3)}{\ln x}\right].
\end{aligned}$$

(4) For $f(x) = (\sqrt{x^2-1})^{\log_3 x}$, logarithmic differentiation is most convenient. Setting $y = (\sqrt{x^2-1})^{\log_3 x} = (x^2-1)^{\frac{1}{2}\log_3 x}$, we have

$$\ln y = \ln\left((x^2-1)^{\frac{1}{2}\log_3 x}\right) = \frac{1}{2}\log_3 x\cdot\ln(x^2-1) = \frac{1}{2\ln 3}\ln x\cdot\ln(x^2-1)$$

by the change of base formula. Differentiating implicitly with respect to x gives

$$\begin{aligned}
\frac{1}{y}\cdot\frac{dy}{dx} &= \left[\frac{1}{2\ln 3}\ln x\cdot\ln(x^2-1)\right]' \\
&= \frac{1}{2\ln 3}\left[(\ln x)'\cdot\ln(x^2-1) + \ln x\cdot[\ln(x^2-1)]'\right] \quad \text{[product rule]} \\
&= \frac{1}{2\ln 3}\left[\frac{1}{x}\cdot\ln(x^2-1) + \ln x\cdot\frac{1}{x^2-1}\cdot 2x\right] \quad \text{[chain rule]} \\
&= \frac{1}{2\ln 3}\left(\frac{\ln(x^2-1)}{x} + \frac{2x\ln x}{x^2-1}\right).
\end{aligned}$$

Therefore, $\frac{dy}{dx} = y\cdot\frac{1}{2\ln 3}\left(\frac{\ln(x^2-1)}{x} + \frac{2x\ln x}{x^2-1}\right)$, i.e.,

$$f'(x) = \frac{1}{2\ln 3}(x^2-1)^{\frac{1}{2}\log_3 x}\left(\frac{\ln(x^2-1)}{x} + \frac{2x\ln x}{x^2-1}\right).$$

Remark. You may distribute $\frac{1}{2\ln 3}$ inside the parenthesis and apply the change of base formula again to obtain

$$f'(x) = (x^2-1)^{\frac{1}{2}\log_3 x}\left(\frac{\log_3(x^2-1)}{2x} + \frac{x\log_3 x}{x^2-1}\right).$$

(5) For $f(x) = \arctan(x^2) + \arctan(\frac{1}{x^2})$, recall that $\frac{d}{dx}(\arctan(x)) = \frac{1}{1+x^2}$, so

$$\begin{aligned}
f'(x) &= \frac{1}{1+(x^2)^2}\cdot(x^2)' + \frac{1}{1+(\frac{1}{x^2})^2}\cdot\left(\frac{1}{x^2}\right)' \quad \text{[chain rule]} \\
&= \frac{1}{1+x^4}\cdot(2x) + \frac{x^4}{x^4+1}\cdot(-2x^{-3}) \quad \text{[power rule]} \\
&= \frac{2x}{1+x^4} + \frac{-2x}{x^4+1} \\
&= 0.
\end{aligned}$$

Remark. This result suggests that $f(x)$ is a constant! Indeed, with some knowledge in trigonometry, one can show that

$$\arctan(t) + \arctan\left(\frac{1}{t}\right) = \begin{cases} \frac{\pi}{2} & \text{if } t > 0, \\ -\frac{\pi}{2} & \text{if } t < 0. \end{cases}$$

Hence $f(x) = \arctan(x^2) + \arctan(\frac{1}{x^2}) = \frac{\pi}{2}$ identically.

Exercise 4

1. critical points:

$$h'(x) = 6x^2 - 6x - 12 = 6(x-2)(x+1)$$

$$x_{cr} = 2; -1$$

2. we do not need $x_{cr} = -1$, since it is not $[0,3]$

$$h(0) = 5 \qquad h(3) = -4$$
$$h(2) = -15$$

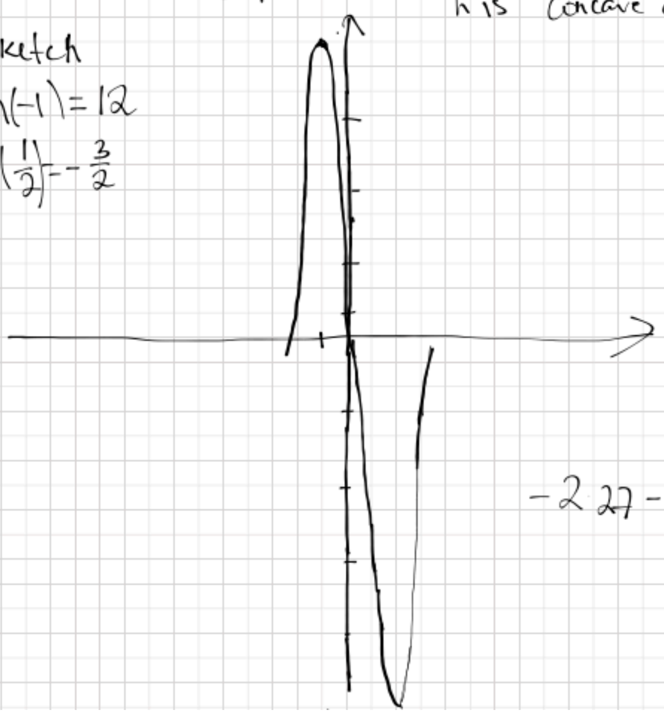
h att. abs. MAX $= 5$ at $x = 0$
and abs. MIN $= -15$ at $x = 2$ on $[0,3]$

2. $+ \quad - \quad +$; $-1 \quad 2$ — h is increas. on $(-\infty, -1) \cup (2; +\infty)$
h is decreas. on $(-1, 2)$

3. $\lim\limits_{x \to +\infty} h(x) = +\infty$;
$\lim\limits_{x \to -\infty} h(x) = -\infty \Rightarrow$ no asymptotes

4. $h''(x) = 12x - 6$
h is concave up on $(\frac{1}{2}, +\infty)$
h is concave down $(-\infty, \frac{1}{2})$

5. Sketch
$h(-1) = 12$
$h(\frac{1}{2}) = -\frac{3}{2}$

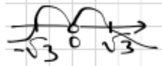


$-2 \; 27 - 27 + 36 \; 15$
$- 3 \cdot 27 + 41$

Exercise 5

1. Domain $(-\infty;0)\cup(0,+\infty)$

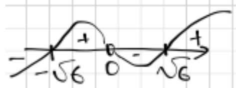
2. $f'(x) = -x^{-2} + 3x^{-4} = \dfrac{3-x^2}{x^4}$



f is decreas. on $(-\infty;\sqrt3)\cup(\sqrt3,+\infty)$

f is increas. on $(-\sqrt3,0)\cup(0,\sqrt3)$

3. $f''(x) = 2x^{-3} - 12x^{-5} = \dfrac{2x^2-12}{x^5}$



f is concave up $(-\sqrt6,0)\cup(\sqrt6,+\infty)$

concave down $(-\infty,-\sqrt6)\cup(0;\sqrt6)$

4. asymptotes

Vertical: $x_0=0$ $\lim\limits_{x\to0^+}\left(\dfrac1x-\dfrac1{x^3}\right)=-\infty$

$\lim\limits_{x\to0^-}\left(\dfrac1x-\dfrac1{x^3}\right)=+\infty$

$x=0$ — vert. as.

Horizontal: $\lim\limits_{x\to+\infty} f(x)=0$

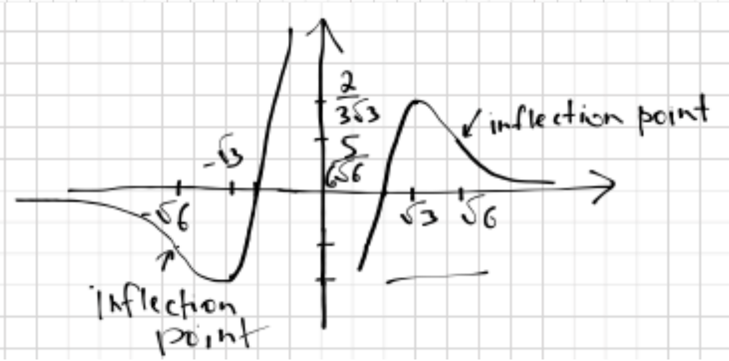
$\lim\limits_{x\to-\infty} f(x)=0$

$y=0$ - horiz. asympt.

5. Sketch: $f(-\sqrt3)=-\dfrac{2}{3\sqrt3}$ $f(\sqrt3)=\dfrac{2}{3\sqrt3}$

$f(1)=0=f(-1)$

$f(\sqrt6)=\dfrac{5}{6\sqrt6}$ $f(-\sqrt6)=-\dfrac{5}{6\sqrt6}$



Problem Set

Exercise 1

Rolle's Theorem
Let f be a function that satisfies the following three hypotheses.
1. f is continuous on the closed interval $[a, b]$.
2. f is differentiable on the open interval (a, b).
3. $f(a) = f(b)$
Then *there is a number c in (a, b) such that $f'(c) = 0$.*

a.

f is a polynomial.

1 and 2: Polynomials are continuous and differentiable everywhere .

3:
$f(3) = 2(3^2) - 4(3) + 5 = 18 - 12 + 5 = 11$

$f(-1) = 2(-1)^2 - 4(-1) + 5 = 2 + 4 + 5 = 11$

All three hypotheses are satisfied. We know that there is a $x = c \in (-1, 3)$ for which $f'(c) = 0$

$f'(x) = 4x - 4 = 0$

$x = c = 1$

b.

f is a polynomial.

1 and 2: Polynomials are continuous and differentiable everywhere .

3:
$f(-2) = (-2)^3 - 2(-2)^2 - 4(-2) + 2 = -8 - 8 + 8 + 2 = -6$

$f(2) = (2)^3 - 2(2)^2 - 4(2) + 2 = 8 - 8 - 8 + 2 = -6$

All three hypotheses are satisfied. We know that there is a $x = c \in (-2, 2)$ for which $f'(c) = 0$

$f'(x) = 3x^2 - 4x - 4 = 0$

c.

$$x_{12} = \frac{4 \pm \sqrt{16 + 48}}{6} = \frac{4 \pm 8}{6}$$

$$x_1 = 2, \quad x_2 = -\frac{2}{3}$$

$x_1 = 2$ does not belong to the interval (-2,2), so

$$x_2 = c = -\frac{2}{3}$$

The sine function is

1. continuous everywhere, (so is f)

2. differentiable everywhere (so is f) .

3.
$f(\pi/2) = \sin(\pi/4) = \sqrt{2}/2.$

$f(3\pi/2) = \sin(3\pi/4) = \sqrt{2}/2$

All three hypotheses are satisfied.
We know that there is an $x = c \in (\pi/2, 3\pi/2)$ for which $f'(c) = 0$

$f'(x)$ =(chain rule)= $\cos(x/2)(\frac{1}{2}) \quad = 0.$

$x/2 = \pi/2 + k\pi \qquad (k \in \mathbb{Z}, \quad$ since cos x is periodic)

$x = \pi + 2k\pi$

In $(\pi/2, 3\pi/2)$, *we have* just $c = \pi$

d.

The discontinuity that f has, at x=0, is in NOT in $[\frac{1}{2}, 2]$. So

1. f is continuous on $[\frac{1}{2}, 2]$,

2. f is differentiable (sum of polynomial and rational function) on $(\frac{1}{2}, 2)$,

3.

$f(\frac{1}{2}) = 2 + \frac{1}{2}$

$f(2) = \frac{1}{2} + 2 = f(\frac{1}{2})$

All three hypotheses are satisfied.
We know that there is an $x = c \in (\frac{1}{2}, 2)$ for which $f'(c) = 0$

$f'(x) = 1 - x^{-2} \qquad = 0$

$x = \pm 1.$

The negative solution is not in the interval, so

$c = 1$

Exercise 2

Part a: $f(x) = x^{2/3}$, $f'(x) = \frac{2}{3} x^{-1/3}$

The linear approx of f at $x = 8$ is

$$y - f(8) = f'(8)(x-8)$$

Subbing in gives

$$y = \frac{1}{3}(x-8) + 4$$

So near $x = 8$, we have that

$$f(8.03) \approx \frac{1}{3}(8.03-8)+4 = \frac{1}{3} \cdot \frac{3}{100} + 4 = 4.01$$

Part b: $f(x) = -x^3 - 2x^2 + x + 3$, $f'(x) = -3x^2 - 4x + 1$

$$x_2 = x_1 - f(x_1)/f'(x_1) = 1 - (1)/(-6) = 7/6$$

$$x_3 = x_2 - f(x_2)/f'(x_2) = 7/6 - \left(\frac{-31}{216}\right)/\left(\frac{-31}{4}\right) = 31/27$$

Part c: The MVT say that there is a value $0 < c < 2$ satisfying

$$f(2) - f(0) = f'(c)(2-0)$$

So plugging in yields

$$f(2) = f(2) - 0 = f(2) - f(0) = f'(c)(2-0) = 2f'(c) \leq 2$$

$$\Rightarrow f(2) \leq 2$$

$$\Rightarrow f(2) \neq 3$$

Exercise 3

Part a: Minimize distance from $(x, \sqrt{x})$ to $(4,0)$

$\Rightarrow$ minimize distance squared from $(x, \sqrt{x})$ to $(4,0)$

$\Rightarrow$ minimize

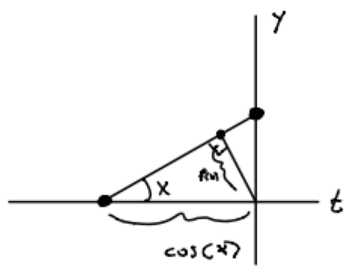
$$d(x) = (x-4)^2 + (\sqrt{x})^2$$
$$= x^2 - 8x + 16 + x$$
$$= x^2 - 7x + 6$$

$0 = d'(x) = 2x - 7 \Rightarrow x = \frac{7}{2}$

$d''(x) = 2 \Rightarrow x = \frac{7}{2}$ is equal to a local minimum

$\Rightarrow$ point is $\left(7/2, \sqrt{7/2}\right)$

Part b: • The distance from the ladder to the origin is



$\sin(x) = f(x)/\cos(x) \Rightarrow f(x) = \sin(x)\cos(x)$

• $f'(x) = \cos(x)\cos(x) - \sin(x)\sin(x) = \cos^2 x - \sin^2 x$

$0 = f'(x) \Rightarrow \cos^2 x = \sin^2 x \Rightarrow \tan^2 x = 1 \Rightarrow x = \pi/4$

$f''(x) = -2\cos(x)\sin(x) - 2\sin(x)\cos(x) = -4\sin(x)\cos(x)$

$\Rightarrow f''(\pi/4) < 0$

$\Rightarrow x = \pi/4$ is maximizer.

Exercise 4

(1) Let $f(x) = x\ln x - 1$. Then $f'(x) = \ln x + x \cdot \frac{1}{x} = \ln x + 1$, so

$$x_{n+1} = x_n - \frac{x_n \ln x_n - 1}{\ln x_n + 1}.$$

For $x_1 = 1$, we get

$$x_2 = 1 - \frac{1\ln 1 - 1}{\ln 1 + 1} = 1 - \frac{-1}{1} = 2$$

and

$$x_3 = 2 - \frac{2\ln 2 - 1}{\ln 2 + 1} \approx 1.7718.$$

Remark. The actual solution to $x\ln x = 1$ is $1.76322\cdots$.

(2) Let $f(x) = x^3 - 2$. Then $f'(x) = 3x^2$, so

$$x_{n+1} = x_n - \frac{x_n^3 - 2}{3x_n^2}.$$

For $x_1 = 1$, we get

$$x_2 = 1 - \frac{1^3 - 2}{3 \cdot 1^2} = 1 - \frac{-1}{3} = \frac{4}{3}$$

and

$$x_3 = \frac{4}{3} - \frac{(\frac{4}{3})^3 - 2}{3(\frac{4}{3})^2} = \frac{4}{3} - \frac{10}{144} = \frac{91}{72} \approx 1.2639.$$

Remark. The actual value of $2^{\frac{1}{3}}$ is $1.25992\cdots$.

Exercise 5

Part (a): $\lim_{x \to +\infty} x - \ln(x) = \lim_{x \to +\infty} \ln(\exp(x - \ln(x)))$

$= \lim_{x \to +\infty} \ln(e^x/x)$

$= \ln\left(\lim_{x \to +\infty} \frac{e^x}{x}\right)$ L'Hopitals rule

$= \ln\left(\lim_{x \to +\infty} e^x/1\right)$

$= \lim_{x \to +\infty} \ln(e^x)$

$= \lim_{x \to +\infty} x$

$= +\infty$

Part (b): $\lim_{x \to +\infty} \frac{e^x}{e^x - e^{-x}} = \lim_{x \to +\infty} \frac{e^{2x}}{e^{2x} - 1} = \lim_{x \to +\infty} \frac{2e^{2x}}{2e^{2x}} = 1$

L'Hopital's Rule

Part (c): $\lim_{x\to 0^+} x^{\frac{\sin(x)}{x\ln(x)}} = \lim_{x\to 0^+} \exp\left(\frac{\sin(x)}{x\ln(x)}\ln(x)\right)$

$= \lim_{x\to 0^+} \exp\left(\frac{\sin(x)}{x}\right)$

$= \exp\left(\lim_{x\to 0^+} \frac{\sin(x)}{x}\right)$

$= \exp(1)$

$= e$

Part (d): $\lim_{x\to 0} \frac{\sin^5 x}{\sin(x^5)} = \lim_{x\to 0} \frac{\sin^5 x}{x^5} \cdot \frac{x^5}{\sin(x^5)}$

$= \lim_{x\to 0} \left(\frac{\sin(x)}{x}\right)^5 \cdot \lim_{x\to 0} \frac{x^5}{\sin(x^5)}$

l'H

$= \left(\lim_{x\to 0} \frac{\sin(x)}{x}\right)^5 \cdot \lim_{x\to 0} \frac{5x^4}{5x^4\cos(x^5)}$

$= 1 \cdot \lim_{x\to 0} \frac{1}{\cos(x^5)}$

$= 1$