Calculus 1–Section 2–Spring 2019–HW8

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Review

Exercise 1

Are the following statements True (T) or False (F)? Explain your choice.

a) If the left and right limits of $f(x)$ exist at a point x_0 then the function $f(x)$ has a limit at x_0.

b) The range of $f(x) = \sqrt{1 - \frac{1}{2}\cos x}$ is $\left[\frac{1}{\sqrt{2}}, \frac{\sqrt{3}}{\sqrt{2}}\right]$.

c) Equation $\cos\left(\frac{\pi}{2} \cdot x\right) = \log_2\left(x - \frac{1}{2}\right)$ has at least one root.

d) Function $g(x)$ given below is continuous on the whole real line for $c = -1$.

$$f(x) = \begin{cases} c \cdot 2^{x^2+1} & x \geq 0; \\ \dfrac{1 - \cos x}{x^2} & x < 0. \end{cases}$$

e) Sequence $a_n = \dfrac{\sqrt{n^2 - 3}}{n + 1}$ converges to 0 as $n \to \infty$. Explain your answer.

f) If a function $f(x)$ is continuous at x_0, it is differentiable at x_0.

Exercise 2

Find the horizontal and vertical asymptotes (if any) of
$$f(x) = \frac{x^2 - x + 3}{1 - x^2}.$$

Exercise 3

Differentiate using any method. Simplify your answer.

- $f(x) = \dfrac{e^{x^2}}{\sin 3x}$;
- $f(x) = \tan x \cdot \log_2^2 x$;
- $f(x) = (\ln(x^2))^{x^5+3x}$;
- $f(x) = (\sqrt{x^2 - 1})^{\log_3 x}$;
- $f(x) = \arctan(x^2) + \arctan(\frac{1}{x^2})$.

Exercise 4

Find the absolute minimum and maximum values of the function

$$h(x) = 2x^3 - 3x^2 - 12x + 5$$

on the interval [0, 3]. Sketch the graph.

Exercise 5

Consider the function

$$f(x) = \frac{1}{x} - \frac{1}{x^3}.$$

1) Find where the function is increasing/decreasing.
2) On what intervals is f(x) concave up/down?
3) Determine if f(x) has any asymptotes.
4) Sketch the graph of f(x).

Problem Set

Exercise 1

Verify that the function satisfies the three hypotheses of Rolle's Theorem on the given interval. Then find all numbers *c* that satisfy the conclusion of Rolle's Theorem.

a. $f(x) = 2x^2 - 4x + 5, \quad [-1, 3]$
b. $f(x) = x^3 - 2x^2 - 4x + 2, \quad [-2, 2]$
c. $f(x) = \sin(x/2), \quad [\pi/2, 3\pi/2]$
d. $f(x) = x + 1/x, \quad \left[\frac{1}{2}, 2\right]$

Exercise 2

- Use linear approximation to estimate $(8.03)^{\frac{2}{3}}$;
- Use Newton's method to approximate the root of the function

$$f(x) = -x^3 - 2x^2 + x + 3$$

  with an initial guess $x_1 = 1$. Produce two successive approximations;
- Let f be a differentiable function such that $f(0) = 0$ and $f'(x) \leq 1$ for all x. Use the MVT to show that $f(2) \neq 3$.

Exercise 3

- Find a point p on the graph of $y = \sqrt{x}$ with the shortest distance to the point $(4,0)$.
- A ladder of length 1 is one side at the wall and one side on the floor. First verify that the distance from the ladder to the corner is $f(x) = \sin x \cos x$, where x is the angle between the floor and the ladder. Find the angle x for which f is maximal.

Exercise 4

- Consider equation $x \ln x = 1$. It has a unique solution. Use Newton's method with the initial guess $x_1 = 1$ to produce two successive approximations to the solution.
- Use Newton's method with the initial guess $x_1 = 1$ to produce two successive approximations to $2^{\frac{1}{3}}$.

Exercise 5

Find each of the following limits, with justification. Indicate the indeterminate forms when they occur. If the function approaches infinity at the point from left or right then explain whether it is +∞ or −∞.

- $\lim\limits_{x \to +\infty} (x - \ln x)$;
- $\lim\limits_{x \to +\infty} \dfrac{e^x}{e^x - e^{-x}}$;
- $\lim\limits_{x \to 0+} x^{\frac{\sin x}{x \ln x}}$;
- $\lim\limits_{x \to 0} \dfrac{\sin^5 x}{\sin(x^5)}$;