Ritt Lectures SPRING 2023

Speaker: Professor Camillo De Lellis (Institute for Advanced Study)
Title: Area-minimizing Integral Currents: Singularities and Structure

"Area-minimizing integral currents are a natural generalization of area-minimizing oriented surfaces. The concept was pioneered by De Giorgi for hypersurfaces of the Euclidean space, and extended by Federer and Fleming to any codimension and general Riemannian ambients. These classical works of the fifties and sixties establish a general existence theory for the *oriented* Plateau problem of finding surfaces of least area spanning a given contour. Celebrated examples of singular 7-dimensional minimizers in R^8 and of singular 2-dimensional minimizers in R^4 are known since long and in fact in these cases there is no *smooth* oriented minimizer and any smooth minimizing sequence converges to the singular ones in an appropriate sense. A first theorem which summarizes the work of several mathematicians in the 60es and 70es (De Giorgi, Fleming, Almgren, Simons, and Federer) and a second theorem of Almgren from 1980 give dimension bounds for the singular set which match the one of the examples, in codimension 1 and in general codimension respectively.

In these lectures I will focus on the case of general codimension and address the question of which structural results can be further proved for the singular set. A recent theorem by Liu proves that the latter canin fact be a fractal of any Hausdorff dimension \alpha \leq m-2. On the other hand it seems likely that it is an(m-2)-rectifiable set, i.e. that it can be covered by countably many C^1submanifolds leaving aside a set of zero (m-2)-dimensional Hausdorff measure. This conjecture is the counterpart, in general codimension, of a celebrated work of Leon Simon in the nineties for the codimension 1 case. In these lectures I will explain why the problem is very challenging, how it can be broken down into easier pieces, and present a line of attack based on recent joint works with Anna Skorobogatova and Paul Minter."

Location: Mathematics Hall, room 520
Monday, April 17th from 4:30 to 5:30pm
Friday, April 21st from 4:45 to 5:45pm