Mathematics V1202
Calculus IV

Midterm Examination #1

October 9, 2006
1:10–2:25 pm

1. Evaluate the iterated integral $\int_0^1 \int_x^1 \cos(y^2)\, dy\, dx$.

2. Find the area of overlap of the two disks bounded by $r = \sin\theta$ and $r = \cos\theta$.

3. Three people, X, Y, and Z, arrive at a meeting during a 1 hour period. Their arrival times are random, independent, and uniform: that is, the probability density is constant. (a) On symmetry grounds, what should be the probability that they arrive in the order X, Y, Z? Why? (b) Set up a triple integral expressing this probability and compute it to get the value you predicted.

4. Sketch carefully, on axes labelled x, y, z, the wedge cut out from the cylinder $x^2+9y^2 = 9$ by the planes $z = 0$ and $z = mx$ for $m > 0$. Then compute its volume.

5. Let $f(x, y)$ be any function with continuous partials, and let $g(x, y) = 2f(x, y)$. If S and T are the surface areas of the graphs of f and g over the unit square $D = [0, 1] \times [0, 1]$, is it necessarily true that $S \le T \le 2S$? Why or why not? State your reasons clearly.

6. Use spherical coordinates to evaluate $\iiint_S (x^2 + y^2 + z^2)^2\, dV$, where S is the solid in the first octant bounded by the sphere $x^2 + y^2 + z^2 = 4$.