

Mathematics V1202
Calculus IV

Midterm Examination #1

October 9, 2006

2:40–3:55 pm

1. Evaluate the iterated integral $\int_0^1 \int_y^1 e^{x^2} dx dy$.
2. (a) Find $\iiint_E \frac{1}{(x^2 + y^2 + z^2)^{n/2}} dV$, where E is the hollow ball bounded by spheres centered at the origin whose radii are r and R , $0 < r < R$.
(b) For what integer values of n does the integral in part (a) have a finite limit as $r \rightarrow 0^+$?
3. Consider the transformation $u = x \cos^2 y$, $v = x \sin^2 y$. For what values of x and y does the Jacobian determinant vanish?
4. Determine the center of mass of a thin C-shaped plate, of constant density, bounded by the circles centered at 0 of radii 1 and 2, and by the lines $y = \pm x$. (See the diagram below.)
5. Find the plane region D for which $\iint_D (x^2 + 4y^2 - 4) dA$ is as small as possible.
6. Let S be the surface area of $z = 2x^2 + 2y^2$ lying over the interior of the ellipse $x^2 + 4y^2 = 4$. Let T be the surface area of $z = 4xy$ lying over the rectangle $[-2, 2] \times [-1, 1]$. Which is greater? Why? (Hint: you shouldn't need to evaluate the integrals.)