Mathematics V1202
Calculus IV

Midterm Examination #2

November 20, 2006
1:10–2:25 pm

PART I: True/False with reason. If true, say so and give a *brief* reason (1 sentence). If false, say so and give a *specific* example where it fails. 6 points each.

1. The vector field shown in the picture below is conservative.

2. If $\mathbf{v}$ is a constant vector field and C a closed smooth parametric curve in $\mathbf{R}^3$, then $\oint_C \mathbf{v} \cdot d\mathbf{r} = 0$.

3. If (P, Q) is a vector field on an open plane region U with $\partial Q/\partial x = \partial P/\partial y$, then line integrals of (P, Q) in U are independent of path.

PART II: Long answers. Give reasons and show all work. 13 points each.

4. Evaluate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$ for the vector field $\mathbf{F}(x, y, z) = (-x, -y, z^2)$, where S is the part of the cone $z = \sqrt{x^2 + y^2}$ between the planes $z = 1$ and $z = 2$ with upward orientation.

5. For any differentiable scalar field f and vector field $\mathbf{G}$ in $\mathbf{R}^3$, show that $\operatorname{curl}(f\mathbf{G}) = (\nabla f) \times \mathbf{G} + f \operatorname{curl} \mathbf{G}$.

6. Evaluate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x, y, z) = (xy^2, yz^2, zx^2 + e^y \sin^7 x)$ and S is the unit sphere, oriented outward.

7. In $\mathbf{R}^3$, let C be the intersection of the cylinder $x^2 + y^2 = 1$ with the graph $z = f(x, y)$ for a differentiable function f. Derive a formula for the arclength of C (as an integral involving the partials of f), and show that it is always at least 2π.