Mathematics V1205y
Calculus IIIS/IVA

Midterm Examination #2

April 17, 2000
9:10–10:35 am

PART I: True/False with reason. If true, say so and give a *brief* reason (1 sentence). If false, say so and give a *specific* example to the contrary. 6 points each.

1. If $\mathbf{F}$ is a smooth vector field, defined on an open connected region, whose line integrals are independent of path, then $\nabla \times \mathbf{F} = 0$.

2. If S is a closed surface and $\mathbf{F}$ is a constant vector field, then $\iint_S \mathbf{F} \cdot d\mathbf{S} = 0$.

3. If $f(x, y, z)$ is a scalar field defined on a convex set with continuous second partials, then $\nabla \cdot (\nabla f) = 0$.

PART II: Long answers. Give reasons and show all work. 13 points each.

4. Let C be the triangle in the plane with vertices $(0, 0)$, $(1, 3)$, and $(0, 3)$, oriented counterclockwise. Let $\mathbf{F} = (xy, \tan y + x^2)$. Compute the line integral $\oint_C \mathbf{F} \cdot d\mathbf{r}$.

5. Let the curve C be the intersection of the hyperbolic paraboloid $z = x^2 - y^2$ and the elliptic cylinder $2x^2 + 3y^2 = 10$, oriented counterclockwise when viewed from above. Let $\mathbf{F} = (2x + ay)\mathbf{i} + (-x + z\cos y)\mathbf{j} + (by + \sin y)\mathbf{k}$. Find values for the constants a and b such that $\oint_C \mathbf{F} \cdot d\mathbf{r} = 0$.

6. Let S be the portion of the paraboloid $z = 1 - x^2 - y^2$ above the plane $z = 0$, oriented upward. Let $\mathbf{F} = \sqrt{1 - z}\,\mathbf{i} - 2e^z\mathbf{k}$. Compute $\iint_S \mathbf{F} \cdot d\mathbf{S}$.

7. Find the curl of the vector field $\mathbf{F} = (yze^{xyz}, xze^{xyz}, xye^{xyz} + \cos z)$. Is it conservative? Why? If it is, find a potential function of which it is the gradient.