1. Compute the divergence and curl of the following vector fields.

\[ \vec{F}(x, y, z) = x^2 \vec{i} + xy \vec{j} + xz \vec{k} \]  \hspace{1cm} (1)

and

\[ \vec{F}(x, y, z) = x \sin z \vec{i} + y \sin x \vec{j} + z \sin y \vec{k} \]  \hspace{1cm} (2)

2. Compute the line integral \( \int_\gamma \vec{F} \cdot d\vec{r} \) where

\[ \vec{F}(x, y) = xy^2 \vec{i} + y^3 \vec{j} \]

and \( \gamma \) is the segment of the curve \( x^2 + 4y^2 = 4 \) that lies in the first quadrant, going from \((0, 1)\) to \((2, 0)\).

3. Compute the line integral \( \int_\gamma \vec{F} \cdot d\vec{r} \) where

\[ \vec{F} = \sin(\cos x) \vec{i} + \left(x^2 + e^{y^3}\right) \vec{j} \]

and \( \gamma \) is the curve given by the segment from \((0, 0)\) to \((1, 1)\) of the curve \( y = x^2 \), followed by a line segment from \((1, 1)\) to \((0, 3)\), and then followed by a line segment from \((0, 3)\) to \((0, 0)\).

4.  
   - Is the following vector field on \( \mathbb{R}^2 \) conservative? If yes, find a potential function for it.

\[ \vec{F}(x, y) = (x^2 - y) \vec{i} + (y^2 - x) \vec{j} \]  \hspace{1cm} (3)

   - What is the value of the line integral \( \int_\gamma \vec{F} \cdot d\vec{r} \) where \( \gamma \) is the curve parametrized by \( r(t) = (t^2, t^3) \) for \( 0 \leq t \leq \sqrt{\pi} \).

5. Find the upward pointing normal vector to the surface \( z - x^2 - y^3 - 1 = 0 \) at the point \((1, 1, 3)\).

6. What is the flux of the vector field \( \vec{F} = -\vec{i} + x \vec{j} + z \sin(y) \vec{k} \) over the part of the surface \( z = x \cos(y) \) that lies above the rectangle \( 0 \leq x \leq 1 \) and \( 0 \leq y \leq \pi \), with upward orientation.
7. Let $C$ and $C'$ be two simple closed curve in the plane that encloses the origin in a counterclockwise direction, show that the integral of the following vector field $\vec{F}$ over $C$ and $C'$ are the same.

$$\vec{F}(x, y) = -\frac{2xy}{(x^2 + y^2)^2}\vec{i} + \frac{x^2 - y^2}{(x^2 + y^2)^2}\vec{j}$$

(Hint: notice that $\vec{F}$ is only defined at $\mathbb{R}^2 - (0, 0)$. Compute $\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}$ and try using Green’s theorem.)